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CHAPTER

4 Rational Expressions

Doug Nance opens the valve to fill his new swimming pool. Without his knowledge, the drain to empty the pool has been left open. If the pool would normally fill in 10 hours and the drain can empty the full pool in 15 hours, how long does it take to fill the pool?

4–1 ■ Fundamental principle of rational expressions

The rational expression

In chapter 1, we defined a rational number to be any number that can be expressed as a quotient of two integers where the denominator is not zero. That is, the set of rational numbers are the elements in the set

$$\left\{\frac{p}{q}\,\middle|\, p \text{ and } q \text{ are integers, } q \neq 0\right\}$$

If this definition is extended to polynomials, the result is called a **rational expression.**

Definition

A **rational expression** is any algebraic expression that can be written as a quotient of two polynomials, where the denominator is not equal to zero.

That is, rational expressions are the elements in the set

$$\left\{\frac{P}{Q}\,\middle|\, P \text{ and } Q \text{ are polynomials, } Q \neq 0\right\}$$

The following are rational expressions:

$$\frac{2x}{3},\quad \frac{2x-3}{x^2+6x-1},\ \frac{5x+1}{3},\quad \text{and}\quad 5x^3-3x+1$$

Note $5x^3 - 3x + 1$ is a rational expression since it can be written

$$\frac{5x^3 - 3x + 1}{1}$$

Recall that division by zero is not defined. The rational expression becomes meaningless for those values of the variable for which the denominator is zero. For example, the rational expression

$$\frac{3x - 2}{x - 6}$$

is meaningless if $x = 6$, because then $x - 6 = 6 - 6 = 0$. Thus we say that

$$\frac{3x - 2}{x - 6}, x \neq 6$$

is a rational expression.

Note The restrictions on the variable are found by setting all factors of the denominator that contain the variable equal to zero and solving the resulting equation(s) for the variable.

Domain of a rational expression

We call the set of all those values that the variable in a rational expression can assume the **domain** of the rational expression.

Definition of the domain

The **domain** of a rational expression is the set of all replacement values of the variable for which the expression is defined.

Thus, given the rational expression

$$\frac{3x - 2}{x - 6}$$

the domain is defined to be the set of all real numbers except 6. In set-builder notation, we symbolize the domain as follows:

$$\text{domain} = \{x | x \in R, x \neq 6\}$$

When the factored denominator consists of more than one factor containing the variable, we use the following property to determine restrictions on the variable.

Zero product property

If P and Q are polynomials, then

$$P \cdot Q = 0$$

if and only if $P = 0$ or $Q = 0$.

Concept

The product of two polynomials is zero provided one or both of the polynomials is equal to zero.

Example 4–1 A

Determine the domain of the following rational expressions. Express this in set-builder notation.

1. $\dfrac{x-5}{x+7}$

$x + 7 = 0$ — Set the denominator equal to 0

$x = -7$ — Solve the equation

The number -7 cannot be used as a replacement for x.

Domain $= \{x \mid x \in R, x \neq -7\}$

2. $\dfrac{4x-3}{x^2-16}$

$x^2 - 16 = (x+4)(x-4)$ — Factor the denominator

$x + 4 = 0$ or $x - 4 = 0$ — Set each factor equal to 0

$x = -4$ $\quad$ $x = 4$

The numbers -4 and 4 cannot be used as replacements for x.

Domain $= \{x \mid x \in R, x \neq -4, x \neq 4\}$

3. $\dfrac{5y^2}{y^2+9}$

Since $y^2 + 9$ is positive for every value of y, then $y^2 + 9$ is *never* equal to zero. Hence, the domain $= \{y \mid y \in R\}$.

▶ ***Quick check*** Determine the domain of $\dfrac{3y-2}{y^2-1}$ and express the domain in set-builder notation.

Reducing to lowest terms

The methods used to simplify rational expressions by reducing them to lowest terms are similar to those used to simplify fractions. A rational expression is reduced to lowest terms when there is no polynomial factor (other than 1 or -1) that is common to both the numerator and the denominator. This is accomplished by factoring the numerator and the denominator and removing a factor of 1. Recall

$$\frac{8}{8} = 1; \quad \frac{-9}{-9} = 1; \quad \frac{x-2}{x-2} = 1, \text{ if } x \neq 2$$

Given polynomials P, Q, and R,

$$\frac{PR}{QR} = \frac{P \cdot R}{Q \cdot R} = \frac{P}{Q} \cdot \frac{R}{R} = \frac{P}{Q} \cdot 1 = \frac{P}{Q}$$

Thus,

$$\frac{(x+2)(x-3)}{(2x-1)(x-3)} = \frac{x+2}{2x-1} \cdot \frac{x-3}{x-3} = \frac{x+2}{2x-1} \cdot 1 = \frac{x+2}{2x-1} \quad \left(x \neq \frac{1}{2} \text{ or } 3\right)$$

We have applied the fundamental principle of rational expressions which is now stated.

Fundamental principle of rational expressions

If P, Q, and R are polynomials, $Q \neq 0$ and $R \neq 0$, then

$$\frac{P}{Q} = \frac{P \cdot R}{Q \cdot R} \quad \text{and} \quad \frac{P \cdot R}{Q \cdot R} = \frac{P}{Q}$$

Concept

Both the numerator and the denominator of a rational expression may be multiplied or divided by the same nonzero polynomial without changing the value of the expression.

By this principle, the rational expression

$$\frac{x-3}{x^2-9} = \frac{x-3}{(x+3)(x-3)}$$ Factor the denominator

$$= \frac{1}{x+3} \quad (x \neq -3, x \neq 3)$$ Divide the numerator and the denominator by $x - 3$

Note The domain of the expression excludes -3 and 3 even though the factor $x - 3$ is not in the resulting expression.

We use the **fundamental principle of rational expressions** to simplify rational expressions by reducing them to lowest terms. This is accomplished by dividing the numerator and the denominator by all common factors.

To reduce a rational expression to lowest terms

1. Completely factor the numerator and the denominator.
2. Divide the numerator and the denominator by the greatest common factor.

Example 4–1 B

Simplify each rational expression by reducing it to lowest terms. State any necessary restrictions on the variables.

Greatest common factor

1. $$\frac{16x^2}{24x^4} = \frac{2(8x^2)}{3x^2(8x^2)}$$ Factor the numerator and the denominator

Greatest common factor

$$= \frac{2}{3x^2} \quad (x \neq 0)$$ Divide the numerator and the denominator by $8x^2$

2. $$\frac{6x-18}{5x-15} = \frac{6(x-3)}{5(x-3)}$$ Factor the numerator and the denominator

$$= \frac{6}{5} \quad (x \neq 3)$$ Divide the numerator and the denominator by the common factor $x - 3$

3. $\dfrac{y-4}{4-y}$

We note that the numerator and the denominator are exactly the same except for the signs. That is, $4 - y = -y + 4 = (-1)(y - 4)$. The given expression can be written in lowest terms by multiplying the numerator and the denominator by -1.

$$= \frac{(-1)(y-4)}{(-1)(4-y)}$$ Multiply numerator and denominator by -1

$$= \frac{(-1)(y-4)}{y-4}$$ Multiply in the denominator, $(-1)(4-y) = y-4$

$$= \frac{-1}{1}$$ Reduce by the common factor $y - 4$

$$= -1 \quad (y \neq 4)$$

4. $$-\frac{x^2-x-12}{20-x-x^2} = -\frac{(x-4)(x+3)}{(4-x)(5+x)}$$ Factor the numerator and the denominator

We note opposite factors $x - 4$ and $4 - x$ in the numerator and the denominator.

$$= -\frac{(-1)(x-4)(x+3)}{(-1)(4-x)(5+x)}$$ Multiply numerator and denominator by -1

$$= -\frac{(-1)(x-4)(x+3)}{(x-4)(5+x)}$$ $(-1)(4-x) = x-4$

$$= -\frac{(-1)(x+3)}{1(5+x)}$$ Reduce by $x - 4$

$$= \frac{x+3}{x+5} \quad (x \neq 4, x \neq -5)$$ $-\frac{-a}{b} = \frac{a}{b}$ and $5 + x = x + 5$

▶ ***Quick check*** Simplify $\dfrac{x^2+3x-28}{x^2-16}$ by reducing to lowest terms. State any necessary restrictions on the variables.

Mastery points

Can you

- Determine the domain of a rational expression?
- Apply the fundamental principle of rational expressions to reduce a rational expression to lowest terms?

Exercise 4–1

State the domain of the given rational expression in set-builder notation. See example 4–1 A.

Example $\dfrac{3y-2}{y^2-1}$

Solution $= \dfrac{3y-2}{(y+1)(y-1)}$ Factor the denominator

$y + 1 = 0$ or $y - 1 = 0$ Set each factor in the denominator equal to 0

$y = -1$ $\quad y = 1$ Solve each equation

The numbers 1 and -1 cannot be used as replacements for y.

Domain $= \{y \mid y \in R, y \neq -1, y \neq 1\}$

1. $\dfrac{3}{x-4}$ 2. $\dfrac{5}{x-8}$ 3. $\dfrac{x}{x+1}$ 4. $\dfrac{5y}{y+9}$ $\boxed{5.}$ $\dfrac{3z}{2z-3}$

6. $\dfrac{5x}{8x-5}$ 7. $\dfrac{x-3}{7x+4}$ 8. $\dfrac{p+9}{6p+5}$ 9. $\dfrac{2x-3}{x^2-4x}$ $\boxed{10.}$ $\dfrac{m+5}{3m^2+6m}$

11. $\dfrac{4x-7}{x^2+8x+16}$ 12. $\dfrac{7-5x}{x^2-14x+49}$ $\boxed{13.}$ $\dfrac{2y-5}{4y^2-25}$ 14. $\dfrac{3x+4}{9x^2-16}$

15. $\dfrac{8x^2+1}{2x^2-5x-3}$ 16. $\dfrac{p^2-7p+1}{4p^2+p-3}$ $\boxed{17.}$ $\dfrac{4x^2-2x+1}{x^2+4}$ 18. $\dfrac{x-3}{x^2+5}$

Simplify the given rational expression by reducing to lowest terms. State any necessary restrictions on the variable. See example 4–1 B.

Example $\dfrac{x^2+3x-28}{x^2-16}$

Solution $= \dfrac{(x-4)(x+7)}{(x-4)(x+4)}$ Factor the numerator and the denominator

Greatest common factor

Greatest common factor

$= \dfrac{x+7}{x+4}\ (x \neq -4)$ Reduce by $x - 4$

19. $\dfrac{25x^3}{15x^4}$ 20. $\dfrac{36y^4}{-42y^2}$ 21. $\dfrac{p^3q^5}{pq^7}$ 22. $\dfrac{-m^5n^3}{-m^8n}$ 23. $\dfrac{a^2b^4c^3}{-abc^7}$

24. $\dfrac{-x^4y^6z}{x^3y^5z}$ 25. $-\dfrac{-27mn^3p^7}{36m^4np^5}$ 26. $\dfrac{72p^3qr^4}{-63pqr}$ 27. $\dfrac{3x-6}{7x-14}$ 28. $\dfrac{8a+12}{6a+9}$

29. $\dfrac{15a}{5a^2-10a}$ $\boxed{30.}$ $\dfrac{-36x}{42x^3+24x}$ 31. $\dfrac{12x^4-12x^3}{6x^3}$ 32. $\dfrac{24x^2-36x}{32x^2}$ 33. $\dfrac{8y^2-8}{6y-6}$

34. $\dfrac{6x+6}{3x-3}$ $\boxed{35.}$ $\dfrac{5x-5y}{y-x}$ 36. $\dfrac{9b-9a}{a-b}$ 37. $\dfrac{a^2-9}{4a+12}$ 38. $\dfrac{6a+3b}{4a^2-b^2}$

39. $\dfrac{4y^2-1}{1+2y}$ 40. $\dfrac{64-49p^2}{7p-8}$ $\boxed{41.}$ $\dfrac{x^3+8}{x+2}$ 42. $\dfrac{a^3-64}{a-4}$ 43. $\dfrac{3x-3y}{2y^3-2x^3}$

44. $\dfrac{5a^3+5b^3}{7b+7a}$ 45. $\dfrac{y^2-49}{y^2+14y+49}$ 46. $\dfrac{a^2-10a+25}{a^2-25}$ 47. $\dfrac{m^2-4m-12}{m^2-m-6}$

48. $\dfrac{n^2-4n+3}{n^2-5n+6}$ 49. $\dfrac{y^2-4y-32}{y^2-y-20}$ 50. $\dfrac{x^2-x-42}{x^2+12x+36}$ 51. $\dfrac{2a^2-3a+1}{2a^2+a-1}$

52. $\dfrac{3b^2-10b+3}{3b^2-7b+2}$ $\boxed{53.}$ $\dfrac{2x^2+3x-9}{4x^2+13x+3}$ 54. $\dfrac{8y^2-22y+5}{4y^2-15y-4}$ 55. $\dfrac{12m^2+17m+6}{8m^2+10m+3}$

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56. $\dfrac{6x^2 + 17x + 7}{12x^2 + 13x - 35}$ 57. $\dfrac{4a^2 + 2a - 12}{6a^2 - 7a - 3}$ 58. $\dfrac{6x^2 - x - 1}{4x^2 + 18x - 10}$ 59. $\dfrac{5y^2 - 10y - 15}{4y^2 - 36}$

60. $\dfrac{7x^2 - 7}{3x^2 - 15x + 12}$ 61. $\dfrac{3a^2 + 16a - 12}{6 - 7a - 3a^2}$ 62. $\dfrac{a^2 + 2ab - 24b^2}{a^2 - 8ab + 16b^2}$ $\boxed{63.}$ $\dfrac{p^2 + 7pq + 12q^2}{p^2 + 5pq + 6q^2}$

Review exercises

Perform the indicated operations. See section 1–4.

1. $\dfrac{3}{4} - \dfrac{5}{6}$ 2. $\dfrac{5}{6} \div \dfrac{10}{12}$ 3. $\dfrac{7}{8} + \dfrac{3}{4}$

Find the solution set of the following equations. See section 2–1.

4. $4(x - 2) + 3 = 2x - 1$ 5. $\dfrac{3}{2}x - 2 = \dfrac{5}{6}x + 1$

6. Solve $4x + 2a = 6x + 5$ for x. See section 2–2.

4–2 ■ Multiplication and division of rational expressions

Multiplication

Recall that the product of two rational numbers is obtained by multiplying the numerators and placing this product over the product of the denominators. This process is indicated by the **multiplication property of rational numbers** stated here.

Multiplication property of rational numbers

If a, b, c, and d are real numbers, $b \neq 0$ and $d \neq 0$, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

To illustrate,

$$\frac{3}{4} \cdot \frac{5}{7} = \frac{3 \cdot 5}{4 \cdot 7} = \frac{15}{28}$$

To multiply two or more rational expressions we use the **multiplication property of rational expressions.**

Multiplication property of rational expressions

If P, Q, R, and S are polynomials, $Q \neq 0$ and $S \neq 0$, then

$$\frac{P}{Q} \cdot \frac{R}{S} = \frac{P \cdot R}{Q \cdot S}$$

Concept

When multiplying two rational expressions, multiply the two numerators to determine the numerator of the product and multiply the two denominators to determine the denominator of the product.

For example,

$$\frac{x-3}{x+1}\cdot\frac{x+2}{x-1}=\frac{(x-3)(x+2)}{(x+1)(x-1)}=\frac{x^2-x-6}{x^2-1}$$

Since we want the resulting product in its simplest form, any possible reduction by common factors should be performed **before** the multiplication of the numerators and the denominators takes place.

To multiply rational expressions

1. Completely factor the numerators and the denominators where possible.
2. Write the factors in the numerators and the denominators as a single rational expression.
3. Divide the numerator and the denominator by all common factors.
4. Multiply the remaining factors in the numerator and in the denominator to obtain a rational expression in lowest terms.

■ *Example 4–2 A*

Find the indicated products reduced to lowest terms. Place restrictions on the variables.

1. $\dfrac{6x^2}{15}\cdot\dfrac{10}{3x^3}=\dfrac{6x^2\cdot 10}{15\cdot 3x^3}$

$=\dfrac{4\cdot 15x^2}{3x\cdot 15x^2}$ — Greatest common factor; Factor numerator and denominator; Greatest common factor

$=\dfrac{4}{3x}\quad(x\neq 0)$ — Divide numerator and denominator by the common factor $15x^2$

2. $\dfrac{3x^2-6x}{x+5}\cdot\dfrac{x^2-25}{x^2-10x+25}$

$=\dfrac{3x(x-2)}{x+5}\cdot\dfrac{(x+5)(x-5)}{(x-5)(x-5)}$ — Factor where possible

$=\dfrac{3x(x-2)(x+5)(x-5)}{(x+5)(x-5)(x-5)}$ — Multiply numerators and denominators

$=\dfrac{3x(x-2)}{x-5}$ — Divide numerator and denominator by the common factors $x+5$ and $x-5$

$=\dfrac{3x^2-6x}{x-5}\quad(x\neq -5,5)$ — Multiply in the numerator

Note You may leave your answer in factored form $\dfrac{3x(x-2)}{x-5}$ or multiply as indicated to obtain $\dfrac{3x^2-6x}{x-5}$, we will show both answers.

3. $\dfrac{3x-1}{x-4}\cdot\dfrac{16-x^2}{3x^2+14x-5}$
$$=\frac{3x-1}{x-4}\cdot\frac{(4-x)(4+x)}{(3x-1)(x+5)}$$
$$=\frac{(3x-1)(4-x)(4+x)}{(x-4)(3x-1)(x+5)}$$

Since there are opposite factors $4-x$ and $x-4$, we multiply the numerator and the denominator by -1.

$$=\frac{(-1)(4-x)(3x-1)(4+x)}{(-1)(x-4)(3x-1)(x+5)}$$
$$=\frac{(-1)(4-x)(3x-1)(4+x)}{(4-x)(3x-1)(x+5)} \qquad (-1)(x-4)=4-x$$
$$=\frac{(-1)(4+x)}{x+5}$$

Reduce by the common factors $4-x$ and $3x-1$

$$=\frac{-x-4}{x+5}\quad\left(x\neq 4,\frac{1}{3},-5\right)$$

▶ ***Quick check*** Find the product of $\dfrac{4x^2-12x}{x+5}\cdot\dfrac{x^2-25}{x^2-6x+9}$ and reduce to lowest terms. Place restrictions on the variable. ■

Division of rational expressions

Recall that the reciprocal of any nonzero real number b is the real number $\dfrac{1}{b}$ found by "inverting" the number. In like fashion, we invert rational expressions to obtain their reciprocals. That is,

$$\frac{x-1}{x+3}\quad\text{and}\quad\frac{x+3}{x-1}\quad\text{are reciprocals.}$$

We use reciprocals in the division of rational expressions as we did when dividing rational numbers.

Division property of rational numbers

If a, b, c, and d are real numbers and $b\neq 0$, $c\neq 0$, and $d\neq 0$, then

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}$$

To illustrate,

$$\frac{5}{8}\div\frac{3}{7}=\frac{5}{8}\cdot\frac{7}{3}=\frac{5\cdot 7}{8\cdot 3}=\frac{35}{24}$$

We can extend this property to rational expressions.

Division property of rational expressions

If P, Q, R, and S are polynomials, $Q\neq 0$, $R\neq 0$, and $S\neq 0$, then

$$\frac{P}{Q}\div\frac{R}{S}=\frac{P}{Q}\cdot\frac{S}{R}=\frac{P\cdot S}{Q\cdot R}$$

Concept

When dividing two rational expressions, the quotient is obtained by multiplying the first expression by the reciprocal of the second expression.

■ Example 4–2 B

Find each of the indicated quotients reduced to lowest terms. Assume no denominator equals zero.

1. $\dfrac{36p^3}{7q} \div \dfrac{15p}{14q^2} = \dfrac{36p^3}{7q} \cdot \dfrac{14q^2}{15p}$ — Multiply by the reciprocal of $\frac{15p}{14q^2}$

$= \dfrac{36 \cdot 14 \cdot p^3q^2}{7 \cdot 15 \cdot pq}$ — Multiply numerators and denominators

$= \dfrac{3 \cdot 12 \cdot 7 \cdot 2 \cdot p^3q^2}{7 \cdot 3 \cdot 5 \cdot pq}$ — Factor numerator and denominator

$= \dfrac{24p^2q \cdot (21pq)}{5 \cdot (21pq)}$ — Greatest common factor; Greatest common factor

$= \dfrac{24p^2q}{5}$ — Reduce by $21pq$

2. $\dfrac{2x+1}{x^2-9} \div \dfrac{2x^2+7x+3}{x-3}$

$= \dfrac{2x+1}{x^2-9} \cdot \dfrac{x-3}{2x^2+7x+3}$ — Multiply by the reciprocal of $\frac{2x^2+7x+3}{x-3}$

$= \dfrac{(2x+1)(x-3)}{(x+3)(x-3)(2x+1)(x+3)}$ — Factor and multiply numerators and denominators

$= \dfrac{1}{(x+3)(x+3)}$ — Reduce by $(x-3)(2x+1)$

$= \dfrac{1}{x^2+6x+9}$ — Multiply as indicated

3. $\dfrac{x^2+2x}{5x} \div (2x^2+x-6)$

Since $2x^2+x-6 = \dfrac{2x^2+x-6}{1}$, then

$\dfrac{x^2+2x}{5x} \div \dfrac{2x^2+x-6}{1} = \dfrac{x^2+2x}{5x} \cdot \dfrac{1}{2x^2+x-6}$ — Multiply by the reciprocal of $\frac{2x^2+6-6}{1}$

$= \dfrac{x(x+2)}{5x(2x-3)(x+2)}$ — Factor and multiply numerators and denominators

$= \dfrac{1}{5(2x-3)}$ — Reduce by $x(x+2)$

$= \dfrac{1}{10x-15}$ — Multiply as indicated

▶ ***Quick check*** Find the quotient of $\dfrac{4x+3}{x^2-49} \div \dfrac{4x^2-13x-12}{x+7}$ and reduce to lowest terms. Assume no denominator equals zero.

Mastery points

Can you
- Multiply rational expressions?
- Divide rational expressions?

Exercise 4–2

Find each indicated product in lowest terms. Place restrictions on the variable. See example 4–2 A.

Example $\dfrac{4x^2-12x}{x+5} \cdot \dfrac{x^2-25}{x^2-6x+9}$

Solution

$= \dfrac{4x(x-3)}{x+5} \cdot \dfrac{(x+5)(x-5)}{(x-3)(x-3)}$ Factor the numerator and the denominator

$= \dfrac{4x(x-3)(x+5)(x-5)}{(x+5)(x-3)(x-3)}$ Multiply the numerators and the denominators

Greatest common factors

$= \dfrac{4x(x-5)(x-3)(x+5)}{(x-3)(x-3)(x+5)}$ Commute

Greatest common factors

$= \dfrac{4x(x-5)}{x-3}$ Reduce by $(x-3)(x+5)$

$= \dfrac{4x^2-20x}{x-3} \quad (x \neq -5,3)$ Multiply in the numerator

1. $\dfrac{8}{9x^2} \cdot \dfrac{12x}{4}$
2. $\dfrac{12b}{7} \cdot \dfrac{28}{4b^3}$
3. $\dfrac{5x^2}{9y^3} \cdot \dfrac{18y}{20x^4}$
4. $\dfrac{15n}{28m} \cdot \dfrac{14m^3}{30n^2}$
5. $\dfrac{16c}{42ab^2} \cdot \dfrac{3a^3b}{8c^4}$
6. $\dfrac{12xy}{25z^3} \cdot \dfrac{5z}{18x^3y^2}$
7. $16a^2b^3 \cdot \dfrac{3}{24ab}$
8. $8x^3yz^2 \cdot \dfrac{15}{36xyz}$
9. $\dfrac{4a+8}{3a-12} \cdot \dfrac{a-2}{5a+10}$
10. $\dfrac{y+3}{8y-16} \cdot \dfrac{3y-6}{4y+12}$
11. $(a+3) \cdot \dfrac{a-6}{6a+18}$
12. $\dfrac{x-3}{5x-25} \cdot (5-x)$
13. $\dfrac{p^2+2p+1}{1-4p} \cdot \dfrac{16p^2-1}{p^2-1}$
14. $\dfrac{m^2-9}{3m+4} \cdot \dfrac{9m^2-16}{m^2+6m+9}$
15. $\dfrac{x^2-9x+20}{x^2-5x+6} \cdot \dfrac{x^2-3x+2}{x^2-5x+4}$
16. $\dfrac{y^2+3y-4}{y^2+2y-3} \cdot \dfrac{y^2-y-6}{y^2+y-12}$
17. $\dfrac{2a^2-a-6}{3a^2-4a+1} \cdot \dfrac{3a^2+7a+2}{2a^2+7a+6}$
18. $\dfrac{4b^2+8b+3}{2b^2-5b-12} \cdot \dfrac{b^2-16}{2b^2+7b+3}$
19. $\dfrac{x^3+8}{x^2-4} \cdot \dfrac{x^2-4x+4}{2x^2+3x-14}$
20. $\dfrac{4x^2-49}{8x^3+27} \cdot \dfrac{4x^2+12x+9}{2x^2-13x+21}$
21. $\dfrac{y-x}{x^2+3xy+2y^2} \cdot \dfrac{x^2+2xy+y^2}{x^2-y^2}$
22. $\dfrac{b^2-a^2}{3a^2+ab-2b^2} \cdot \dfrac{6a^2-ab-2b^2}{a^2-2ab+b^2}$

Find each of the indicated quotients in lowest terms. Assume all denominators are nonzero. See example 4–2 B.

Example $\dfrac{4x+3}{x^2-49} \div \dfrac{4x^2-13x-12}{x+7}$

Solution

$$= \frac{4x+3}{x^2-49} \cdot \frac{x+7}{4x^2-13x-12}$$ Multiply by the reciprocal

$$= \frac{(4x+3)(x+7)}{(x+7)(x-7)(4x+3)(x-4)}$$ Factor the denominator and multiply

$$= \frac{(4x+3)(x+7)}{(4x+3)(x+7)(x-7)(x-4)}$$ Commute

$$= \frac{1}{(x-7)(x-4)}$$ Reduce by GCF $(4x+3)(x+7)$

$$= \frac{1}{x^2-11x+28}$$ Multiply in the denominator

23. $\dfrac{9a^2b}{8ab^3} \div \dfrac{18ab^3}{16a^2b^2}$

24. $\dfrac{7ab^3}{10x^2y^3} \div \dfrac{21a^2b^2}{15x^3y^2}$

25. $\dfrac{r+4}{r^2-1} \div \dfrac{r^2-16}{r+1}$

26. $\dfrac{4x+4y}{x-3} \div \dfrac{x+y}{x^2-9}$

27. $\dfrac{6-2x}{2x+8} \div (9-3x)$

28. $\dfrac{x^2-25}{2x+10} \div (x^2-10x+25)$

29. $(x^2-2x-3) \div \dfrac{4x-12}{x^2-1}$

30. $(4x^2-9) \div \dfrac{4x+6}{x+3}$

31. $\dfrac{x^3-125}{2x+1} \div \dfrac{x^2-25}{2x^2-9x-5}$

32. $\dfrac{3x^2+10x-8}{5x-15} \div \dfrac{x^3+64}{x^2-9}$

33. $\dfrac{56-x-x^2}{x^2-6x-7} \div \dfrac{x^2+4x-21}{x^2-4x-5}$

34. $\dfrac{a^2+16a+63}{20-a-a^2} \div \dfrac{a^2-5a-84}{a^2-a-12}$

35. $\dfrac{2x^2+5x-12}{3x^2-8x-16} \div \dfrac{2x^2+3x-9}{3x^2+13x+12}$

36. $\dfrac{6x^2-5x-4}{8x^2+10x+3} \div \dfrac{3x^2+14x-24}{4x^2-12x-7}$

Find each indicated product or quotient in lowest terms. Assume all denominators are nonzero. See examples 4–2 A and B.

37. $\dfrac{a^2-b^2}{2a+4b} \cdot \dfrac{a+2b}{a+b}$

38. $\dfrac{4x+4y}{x-3y} \cdot \dfrac{x^2-9y^2}{x+y}$

39. $\dfrac{n^2-m^2}{2m-3n} \div \dfrac{m-n}{4m^2-9n^2}$

40. $\dfrac{p^2-q^2}{q-3p} \div \dfrac{p^2+2pq+q^2}{9p^2-q^2}$

41. $\dfrac{a^2-8a+15}{a^2+7a+6} \cdot \dfrac{a^2-6a-7}{a^2-9} \div \dfrac{a^2-12a+35}{a^2-36}$

42. $\dfrac{x^2+3x-28}{x^2-8x+7} \div \dfrac{x^2-7x+12}{x^2-x-42} \cdot \dfrac{2x-8}{x^2-36}$

43. $\dfrac{2x^2+5x-3}{3x^2-10x-8} \div \dfrac{2x^2-7x+3}{3x^2-x-2} \cdot \dfrac{x^2-6x+9}{x^2-2x+1}$

44. $\dfrac{2x^2+x-6}{5x^2+7x-6} \cdot \dfrac{20x^2-7x-3}{6x^2-25x+4} \div \dfrac{4x^2-11x-3}{6x^2-19x+3}$

45. $\dfrac{27y^3-8x^3}{x+y} \div \dfrac{2x-3y}{x^3+y^3}$

46. $\dfrac{a^3+64b^3}{16b^2-a^2} \cdot \dfrac{a+4b}{a^2+3ab-4b^2}$

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$\boxed{47.}$ $\dfrac{ab - a + 3b - 3}{ab + a - 4b - 4} \div \dfrac{ab - 2a + 3b - 6}{ab + 2a - 4b - 8}$

48. $\dfrac{mn - n + 4m - 4}{mn - 3n + 4m - 12} \cdot \dfrac{mn - 3n + 2m - 6}{mn + 2m + n + 2}$

49. $\dfrac{xz - xw + yz - yw}{xz + xw - yz - yw} \cdot \dfrac{xz - xw - yz + yw}{xz + xw + yz + yw}$

50. $\dfrac{pr + ps + qr + qs}{pr - ps - qr + qs} \div \dfrac{xr + xs - yr - ys}{xr - xs + yr - ys}$

Review exercises

Add or subtract the following fractions. See section 1–4.

1. $\dfrac{11}{12} + \dfrac{7}{8}$

2. $\dfrac{14}{15} - \dfrac{3}{5}$

Find the solution set of the following inequalities. See section 2–5.

3. $5y - 2 \leq y + 6$

4. $-7 \leq 2y - 1 < 1$

Find the following products. See section 3–2.

5. $4x^3(2x^2 - x + 6)$

6. $(4y + 1)(y - 4)$

4–3 ■ Addition and subtraction of rational expressions

When we add or subtract a pair of rational numbers that have the same denominator, such as $\dfrac{3}{4} + \dfrac{5}{4}$, we use the property of adding or subtracting fractions.

Property of adding or subtracting rational numbers

If a, b, and c are real numbers, $b \neq 0$, then

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$$

For example,

$$\frac{3}{4} + \frac{5}{4} = \frac{3 + 5}{4} = \frac{8}{4} = 2 \text{ and } \frac{7}{8} - \frac{1}{8} = \frac{7 - 1}{8} = \frac{6}{8} = \frac{3}{4}$$

We can extend this property to rational expressions.

Property of adding or subtracting rational expressions

If P, Q, and R are polynomials, $Q \neq 0$, then

$$\frac{P}{Q} + \frac{R}{Q} = \frac{P + R}{Q} \quad \text{and} \quad \frac{P}{Q} - \frac{R}{Q} = \frac{P - R}{Q}$$

Concept

To add, or subtract, two rational expressions having the same denominators, add, or subtract, the numerators and place this sum or difference over the common denominator.

Example 4-3 A

Find each indicated sum or difference in lowest terms. Assume all denominators are nonzero.

1. $\dfrac{3x+2}{x-2}+\dfrac{4x-5}{x-2}$

$$= \frac{(3x+2)+(4x-5)}{x-2} \quad \text{Add the numerators}$$
$$= \frac{3x+2+4x-5}{x-2} \quad \text{Remove parentheses}$$
$$= \frac{7x-3}{x-2} \quad \text{Combine like terms in the numerator}$$

2. $\dfrac{3x-2y}{x^2-y^2}-\dfrac{2x-3y}{x^2-y^2}$

$$= \frac{(3x-2y)-(2x-3y)}{x^2-y^2} \quad \text{Subtract the numerators}$$
$$= \frac{3x-2y-2x+3y}{x^2-y^2} \quad \text{Remove parentheses}$$
$$= \frac{x+y}{x^2-y^2} \quad \text{Combine like terms}$$
$$= \frac{x+y}{(x+y)(x-y)} \quad \text{Factor the denominator}$$
$$= \frac{1}{x-y} \quad \text{Reduce by } (x+y) \text{ (always reduce where possible)}$$

Note When we write the sum or the difference of the numerators, it is important that we write each numerator in parentheses to avoid making a common mistake when we subtract. Failure to do this would have caused the numerator in example 2 to become $3x-2y-2x-3y=x-5y$.

▶ ***Quick check*** Find the difference of $\dfrac{4x-3}{x^2-4}-\dfrac{3x-1}{x^2-4}$ in lowest terms. Assume the denominator is nonzero.

When adding, or subtracting, two rational expressions $\dfrac{P}{Q}$ and $\dfrac{R}{S}$ that *do not* have the same denominator, we can add or subtract them in the following way:

Property of adding or subtracting rational expressions with different denominators

If P, Q, R, and S are polynomials, $Q \neq 0$ and $S \neq 0$, then

$$\frac{P}{Q}+\frac{R}{S}=\frac{P\cdot S+Q\cdot R}{Q\cdot S} \quad \text{and} \quad \frac{P}{Q}-\frac{R}{S}=\frac{P\cdot S-Q\cdot R}{Q\cdot S}$$

The following examples illustrate the use of this property for rational expressions.

1. $$\frac{x+1}{x-2}+\frac{x-3}{x+4}=\frac{(x+1)(x+4)+(x-2)(x-3)}{(x-2)(x+4)}$$
$$=\frac{(x^2+5x+4)+(x^2-5x+6)}{(x-2)(x+4)}$$ Multiply as indicated
$$=\frac{2x^2+10}{(x-2)(x+4)}\quad(x\neq 2, x\neq -4)$$ Combine like terms

2. $$\frac{x}{x-7}-\frac{2x+1}{x-9}=\frac{x\cdot(x-9)-(x-7)(2x+1)}{(x-7)(x-9)}$$
$$=\frac{(x^2-9x)-(2x^2-13x-7)}{(x-7)(x-9)}$$ Multiply as indicated
$$=\frac{x^2-9x-2x^2+13x+7}{(x-7)(x-9)}$$ Remove parentheses and change signs
$$=\frac{-x^2+4x+7}{(x-7)(x-9)}\quad(x\neq 7, x\neq 9)$$ Combine like terms

Note We do not multiply in the denominator to aid in recognizing common factors for reducing to lowest terms. Always check the numerator for common factors.

This method is useful when adding or subtracting only two rational expressions and when the denominators have no common factors. When rational numbers or rational expressions do not have the same denominator, we apply the fundamental principle of rational expressions to obtain equivalent expressions (expressions that name the same number) with a common denominator. To do this, we must find the *least common multiple* (LCM) of the denominators.

Least common multiple

The least common multiple (LCM) of a set of expressions is the least expression that is exactly divisible by the expressions.

We use the following procedure to find the LCM of a set of polynomials.

Finding the LCM of a set of polynomials

1. Completely factor the polynomials.
2. The LCM consists of the product of all distinct factors of the polynomials, each raised to the greatest power to which it appears in any one of the factorizations.

Example 4–3 B

Find the least common multiple.

1. $24x^3$, $15x^2$, $10x$

$$\left.\begin{aligned}24x^3&=2^3\cdot 3\cdot x^3\\15x^2&=3\cdot 5\cdot x^2\\10x&=2\cdot 5\cdot x\end{aligned}\right\}$$ Completely factor each polynomial

Choose the greatest power of each different factor.

The LCM $=2^3\cdot 3\cdot 5\cdot x^3=120x^3$.

2. $2x - 4,\ x^2 - 4,\ x^2 + 4x + 4$

$$\left.\begin{aligned} 2x - 4 &= 2 \cdot (x - 2) \\ x^2 - 4 &= (x - 2) \cdot (x + 2) \\ x^2 + 4x + 4 &= (x + 2)^2 \end{aligned}\right\}$$

Choose the greatest power of each different factor.

The LCM $= 2(x - 2)(x + 2)^2$.

▶ ***Quick check*** Find the LCM of $3x + 9,\ x^2 - 9,\ x^2 - 6x + 9$.

We now apply the fundamental principle of rational expressions to "build" a given expression to an equivalent rational expression. To do this, we must **multiply** the numerator and the denominator by the same nonzero polynomial.

■ Example 4–3 C

Write each rational expression as an equivalent expression with the indicated denominator. Assume all denominators are nonzero.

1. $\dfrac{13}{3a}$, denominator $12a^2$

We obtain the factor with which to build by dividing $\dfrac{12a^2}{3a} = 4a$.

$$\begin{aligned} \frac{13}{3a} &= \frac{13 \cdot 4a}{3a \cdot 4a} \\ &= \frac{52a}{12a^2} \end{aligned}$$

2. $\dfrac{3x - 2}{2x - 1}$, denominator $4x^2 - 1$

Since $4x^2 - 1$ factors to $(2x - 1)(2x + 1)$, the building factor is $2x + 1$.

$$\begin{aligned} \frac{3x - 2}{2x - 1} &= \frac{(3x - 2)(2x + 1)}{(2x - 1)(2x + 1)} \\ &= \frac{6x^2 - x - 2}{4x^2 - 1} \end{aligned}$$

3. $\dfrac{3}{1 - x}$, denominator $x - 1$

Since $x - 1 = (-1)(1 - x)$, multiply the numerator and the denominator by -1.

$$\frac{3}{1 - x} = \frac{3(-1)}{(1 - x)(-1)} = \frac{-3}{x - 1}$$

▶ ***Quick check*** Write $\dfrac{5z - 3}{3z + 4}$ as an equivalent expression with $9z^2 - 16$ as a denominator. Assume the denominator is nonzero.

We now add and subtract rational expressions that have unlike denominators by building the expressions to equivalent rational expressions having a common denominator—the LCM of the denominators (called the least common denominator, LCD).

■ Example 4–3 D

Find the indicated sum or difference in lowest terms. Assume no denominator equals zero.

1. $\dfrac{6}{5a} + \dfrac{7}{6b}$

The LCM of $5a$ and $6b$ is $5a \cdot 6b = 30ab$.

Building factors

$$= \frac{6}{5a} \cdot \frac{6b}{6b} + \frac{7}{6b} \cdot \frac{5a}{5a} \qquad \text{Multiply by building factors } 6b \text{ and } 5a$$

$$= \frac{36b}{30ab} + \frac{35a}{30ab} \qquad \text{Equivalent expressions with LCD } 30ab$$

$$= \frac{36b + 35a}{30ab} \qquad \text{Add the numerators}$$

2. $\dfrac{5}{16a} - \dfrac{7}{24a^2}$

$$\left.\begin{aligned} 16a &= 2^4 \cdot a \\ 24a^2 &= 2^3 \cdot 3 \cdot a^2 \end{aligned}\right\} \quad \text{The LCM is } 2^4 \cdot 3 \cdot a^2 = 48a^2.$$

Building factors

$$= \frac{5}{16a} \cdot \frac{3a}{3a} - \frac{7}{24a^2} \cdot \frac{2}{2} \qquad \text{Multiply by building factors } 3a \text{ and } 2$$

$$= \frac{15a}{48a^2} - \frac{14}{48a^2} \qquad \text{Multiply as indicated}$$

$$= \frac{15a - 14}{48a^2} \qquad \text{Subtract the numerators}$$

3. $\dfrac{10}{x-3} + \dfrac{9}{x^2 + 3x - 18}$

$$\left.\begin{aligned} x - 3 &= (x-3) \\ x^2 + 3x - 18 &= (x-3)(x+6) \end{aligned}\right\} \quad \text{The LCM is } (x-3)(x+6).$$

$$= \frac{10}{x-3} + \frac{9}{(x-3)(x+6)} \qquad \text{Factor denominators}$$

Building factor

$$= \frac{10}{x-3} \cdot \frac{x+6}{x+6} + \frac{9}{(x-3)(x+6)} \qquad \text{Multiply by building factor } x + 6$$

$$= \frac{10(x+6)}{(x-3)(x+6)} + \frac{9}{(x-3)(x+6)}$$

$$= \frac{10x + 60 + 9}{(x-3)(x+6)} \qquad \text{Add the numerators}$$

$$= \frac{10x + 69}{(x-3)(x+6)} \qquad \text{Combine like terms}$$

4. $\dfrac{y+3}{y^2 + 4y - 21} - \dfrac{y+5}{y^2 + 6y - 27}$

$$\left.\begin{aligned} y^2 + 4y - 21 &= (y-3)\,(y+7) \\ y^2 + 6y - 27 &= (y+9)\,(y-3) \end{aligned}\right\} \quad \text{The LCM is } (y+9)\,(y-3)\,(y+7).$$

$$= \frac{y+3}{(y-3)(y+7)} - \frac{y+5}{(y+9)(y-3)}$$ Factor denominators

Building factors

$$= \frac{y+3}{(y-3)(y+7)} \cdot \frac{y+9}{y+9} - \frac{y+5}{(y+9)(y-3)} \cdot \frac{y+7}{y+7}$$

$$= \frac{(y+3)(y+9)}{(y-3)(y+7)(y+9)} - \frac{(y+5)(y+7)}{(y+9)(y-3)(y+7)}$$ Multiply in numerator and denominator

$$= \frac{(y^2+12y+27)}{(y-3)(y+7)(y+9)} - \frac{(y^2+12y+35)}{(y+9)(y-3)(y+7)}$$ Multiply in the numerators

$$= \frac{(y^2+12y+27)-(y^2+12y+35)}{(y-3)(y+9)(y+7)}$$ Subtract the numerators

$$= \frac{y^2+12y+27-y^2-12y-35}{(y-3)(y+9)(y+7)}$$ Remove parentheses and change signs

$$= \frac{-8}{(y-3)(y+9)(y+7)}$$ Combine like terms

5. $\dfrac{x^2}{x-8} + \dfrac{7x+8}{8-x}$

Since the denominators are the opposites of each other, we multiply the numerator and the denominator of $\dfrac{7x+8}{8-x}$ by -1.

$$\frac{x^2}{x-8} + \frac{7x+8}{8-x} \cdot \frac{-1}{-1} = \frac{x^2}{x-8} + \frac{-7x-8}{x-8}$$ Multiply as indicated

$$= \frac{x^2-7x-8}{x-8}$$ Add numerators

$$= \frac{(x+1)(x-8)}{x-8}$$ Factor the numerator

$$= x+1$$ Reduce by $x-8$

► ***Quick check*** Find the difference $\dfrac{2y+5}{y-2} - \dfrac{4y+12}{y^2+y-6}$ in lowest terms. Assume no denominator equals zero. ■

We now generalize the procedure for adding or subtracting rational expressions.

To add or subtract rational expressions

1. If the denominators are like, add or subtract the numerators and place the result over the common denominator.
2. If the denominators are different, find the least common multiple (LCM) of the denominators.
3. Multiply the numerator and the denominator of each rational expression by all factors present in the LCM but missing in the denominator of the particular rational expression to obtain equivalent rational expressions.
4. Proceed as in step 1. Reduce the results to lowest terms to simplify the answer.

Mastery points

Can you

- Find the least common multiple (LCM) of a set of polynomials?
- Build rational expressions to equivalent rational expressions?
- Add or subtract rational expressions that have the same denominators?
- Add or subtract rational expressions that have different denominators?

Exercise 4–3

Find the least common multiple of the set of polynomials. See example 4–3 B.

Example $3x + 9, x^2 - 9, x^2 - 6x + 9$

Solution
$$\left.\begin{aligned} 3x + 9 &= 3(x + 3) \\ x^2 - 9 &= (x + 3)(x - 3) \\ x^2 - 6x + 9 &= (x - 3)^2 \end{aligned}\right\} \text{Factor the denominators}$$

The LCM is $3(x + 3)(x - 3)^2$.

1. $14x, 35$
2. $42y, 36$
3. $10k, 16k^2$
4. $18x^3, 21x^2$
5. $32a^3b, 9ab^2$
6. $48x^2y^2, 30x^4y$
7. $xy^3, 6x^2y^2, 15xy^4$
8. $4mn^4, 14m^2n^3, 35mn$
9. $4a, 2a - 4$
10. $9b, 15b + 30$
11. $x - 7, 3x - 21, 6x$
12. $x + 9, 4x + 36, 8x$
13. $p^2 - p - 12, p^2 + 6p + 9, 3p - 12$
14. $n^2 - 9, n^2 - n - 6, 8n^2 + 24n$
15. $a^2 - 25, 10a + 50, 5a - 25$
16. $x^2 + 6x + 9, 2x + 6, x^2 + x - 6$
17. $q^2 - 49, q^2 + 5q - 14, q - 7$
18. $2y + 10, y^2 - 25, y^2 - 10y + 25$
19. $a - b, a^2 - b^2, 5a + 5b$
20. $m - n, 4m + 4n, m^2 - 2mn + n^2$
21. $2a^2 - 13a - 7, 6a^2 + a - 1, a^2 - 49$
22. $4m^2 + 5m - 6, 3m^2 + m - 10, 16m^2 - 9$

Build the given rational expression to equivalent rational expression with the given denominator. Assume all denominators are nonzero. See example 4–3 C.

Example $\dfrac{5z - 3}{3z + 4}$, denominator $9z^2 - 16$

Solution Since $9z^2 - 16 = (3z + 4)(3z - 4)$, the building factor is $3z - 4$.

$$= \frac{(5z - 3)(3z - 4)}{(3z + 4)(3z - 4)} \quad \text{Multiply numerator and denominator by } 3z - 4$$

$$= \frac{15z^2 - 29z + 12}{9z^2 - 16} \quad \text{Multiply in numerator and denominator}$$

23. $\dfrac{4}{7x}$, denominator $21x^3$
24. $\dfrac{8}{9y^2}$, denominator $72y^5$
25. $\dfrac{5x}{8y}$, denominator $24x^2y^2$
26. $\dfrac{-3a}{7b^2}$, denominator $35a^2b^3$

27. $4p$, denominator $p - 3$

28. $a + 3$, denominator $2a - 1$

29. $\dfrac{p-3}{p+2}$, denominator $p^2 - 4$

30. $\dfrac{n+7}{n-9}$, denominator $n^2 - 81$

31. $\dfrac{4x}{2x-3}$, denominator $8x^3 - 27$

32. $\dfrac{9x}{3x+1}$, denominator $27x^3 + 1$

33. $\dfrac{2n-1}{n+7}$, denominator $n^2 + 2n - 35$

34. $\dfrac{4x+3}{x-9}$, denominator $x^2 - 4x - 45$

35. $\dfrac{2y-5}{4y-1}$, denominator $4y^2 + 7y - 2$

36. $\dfrac{3x+4}{2x-9}$, denominator $8x^2 - 30x - 27$

37. $-\dfrac{3m}{4-5m}$, denominator $25m^2 - 16$

38. $\dfrac{9-a}{4-a}$, denominator $a^2 + a - 20$

39. $\dfrac{-9}{9-a}$, denominator $a - 9$

40. $-\dfrac{-12}{10-b}$, denominator $b - 10$

Find each indicated sum or difference in lowest terms. Assume all denominators are nonzero. See example 4–3 A.

Example $\dfrac{4x-3}{x^2-4} - \dfrac{3x-1}{x^2-4}$

Solution

$$= \frac{(4x-3)-(3x-1)}{x^2-4} \qquad \text{Subtract numerators}$$

$$= \frac{4x-3-3x+1}{x^2-4} \qquad \text{Remove parentheses and subtract (change signs)}$$

$$= \frac{x-2}{x^2-4} \qquad \text{Combine like terms}$$

$$= \frac{x-2}{(x-2)(x+2)} \qquad \text{Factor the denominator}$$

$$= \frac{1}{x+2} \qquad \text{Reduce by } (x-2)$$

41. $\dfrac{5}{q} + \dfrac{8}{q}$

42. $\dfrac{7}{x} - \dfrac{17}{x}$

43. $\dfrac{4y}{y+4} - \dfrac{7y}{y+4}$

44. $\dfrac{8a}{a-5} + \dfrac{7}{a-5}$

45. $\dfrac{5y}{y+2} + \dfrac{10}{y+2}$

46. $\dfrac{3x}{x-4} - \dfrac{12}{x-4}$

47. $\dfrac{7x-1}{3x+4} - \dfrac{4x-5}{3x+4}$

48. $\dfrac{3y-2}{4y+3} + \dfrac{5y+8}{4y+3}$

49. $\dfrac{3x-4}{x^2+5x+6} - \dfrac{2x-6}{x^2+5x+6}$

50. $\dfrac{2b+1}{b^2-4} + \dfrac{1-b}{b^2-4}$

51. $\dfrac{2x-y}{x+y} - \dfrac{x-2y}{x+y}$

52. $\dfrac{3x-2y}{2x+y} - \dfrac{x+y}{2x+y}$

Find the indicated sum or difference. Assume all denominators are nonzero. See example 4–3 D.

Example $\dfrac{2y+5}{y-2} - \dfrac{4y+12}{y^2+y-6}$

Solution

$$\left.\begin{aligned} y-2 &= (y-2) \\ y^2+y-6 &= (y-2)(y+3) \end{aligned}\right\} \quad \text{Factor denominators}$$

The LCM of the denominators is $(y-2)(y+3)$.

$$= \frac{2y+5}{y-2} - \frac{4y+12}{(y-2)(y+3)} \qquad \text{Factor denominators}$$

$$= \frac{2y+5}{y-2}\cdot\underbrace{\frac{y+3}{y+3}}_{\text{Building factor}} - \frac{4y+12}{(y-2)(y+3)} \qquad \text{Multiply by building factor}$$

$$= \frac{(2y+5)(y+3)}{(y-2)(y+3)} - \frac{4y+12}{(y-2)(y+3)} \qquad \text{Multiply as indicated}$$

$$= \frac{2y^2+11y+15}{(y-2)(y+3)} - \frac{4y+12}{(y-2)(y+3)}$$

$$= \frac{(2y^2+11y+15)-(4y+12)}{(y-2)(y+3)} \qquad \text{Subtract numerators}$$

$$= \frac{2y^2+11y+15-4y-12}{(y-2)(y+3)} \qquad \text{Remove parentheses and subtract (change signs)}$$

$$= \frac{2y^2+7y+3}{(y-2)(y+3)} \qquad \text{Combine like terms}$$

$$= \frac{(2y+1)(y+3)}{(y-2)(y+3)} \qquad \text{Factor numerator}$$

$$= \frac{2y+1}{y-2} \qquad \text{Reduce by } (y+3)$$

53. $\dfrac{8}{3x} + \dfrac{5}{4x}$

54. $\dfrac{7}{5a} - \dfrac{9}{6a}$

55. $\dfrac{5}{z} - \dfrac{2}{z^2}$

56. $\dfrac{7}{y^2} + \dfrac{2}{3y}$

57. $\dfrac{6}{3x} + \dfrac{5}{9x^2}$

58. $\dfrac{8}{4x^2} - \dfrac{3}{2x}$

59. $\dfrac{2}{3x^2y} - \dfrac{4}{9xy^2}$

60. $\dfrac{7}{2ab^2} + \dfrac{6}{6a^2b}$

61. $\dfrac{4a}{3a+5} + \dfrac{2a}{2a-3}$

62. $\dfrac{5m}{m-8} - \dfrac{3m-8}{2m+7}$

63. $\dfrac{17}{5y-10} + \dfrac{19}{2y+4}$

64. $\dfrac{10}{4a-6} - \dfrac{13}{3a+9}$

65. $\dfrac{x+2}{x-9} - \dfrac{x-6}{9-x}$

66. $\dfrac{2a-1}{2a-3} + \dfrac{4-a}{3-2a}$

67. $\dfrac{7}{x^2-5x-6} + \dfrac{9}{x^2-1}$

68. $\dfrac{14}{a^2-7a-18} - \dfrac{8}{a^2-4}$

69. $\dfrac{5x}{x^2-2xy-3y^2} - \dfrac{2y}{x^2+2xy+y^2}$

70. $\dfrac{8q}{4q^2-9p^2} + \dfrac{5q}{4q^2-12pq+9p^2}$

71. $\dfrac{a-7}{2a^2+9a-5} + \dfrac{4-a}{4a^2+23a+15}$

72. $\dfrac{b+9}{12b^2-5b-2} - \dfrac{8-2b}{3b^2-17b+10}$

73. $(4a-3) - \dfrac{2a+5}{5a-2}$

74. $\dfrac{5x+4}{3x+1} + (8x-5)$

75. $\dfrac{5}{8p} + \dfrac{6p-5}{4p^2-8p-60}$

76. $\dfrac{9p-2}{2p^2-2p-84} + \dfrac{7}{6p}$

77. $\dfrac{3y}{y^2 + 5y + 6} - \dfrac{5}{4 - y^2}$

78. $\dfrac{6a}{6 + a - a^2} - \dfrac{7a}{a^2 + 7a + 10}$

79. $\dfrac{b + 3}{b + 2} + \dfrac{2b}{5b^2 - 20}$

80. $\dfrac{3}{6b^2 - 4bc} - \dfrac{4}{9bc - 6c^2}$

81. $\dfrac{2x - 3y}{x^2 - 4xy - 12y^2} - \dfrac{2y - x}{x^2 - 12xy + 36y^2}$

$\boxed{82.}$ $\dfrac{a - b}{a^2 - 3ab - 4b^2} + \dfrac{2b - 5a}{a^2 - 16b^2}$

83. $\dfrac{x - 3}{8x^2 - 26x + 15} + \dfrac{3x + 2}{6x^2 - 13x - 5}$

84. $\dfrac{2p - 3}{8p^2 - 18p - 5} - \dfrac{5 - 7p}{4p^2 - 27p - 7}$

85. $\dfrac{5m - n}{8m^2 + 15mn - 2n^2} + \dfrac{3m + n}{5m^2 + 6mn - 8n^2}$

86. $\dfrac{b - 2a}{3a^2 - 2ab - 8b^2} - \dfrac{3a - 5b}{9a^2 + 6ab - 8b^2}$

Solve the following word problems.

87. Workers A, B, C, and D can complete a given job in p, q, r, and s hours, respectively. Working together they can complete in one hour

$$\frac{1}{p} + \frac{1}{q} + \frac{1}{r} + \frac{1}{s}$$

of the job. Obtain a single expression for what they can do together in one hour.

88. In electricity, the total resistance of any parallel circuit is given by

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

Combine the fractions in the right member.

89. Given the focal lengths f_1 and f_2 of two thin lenses that are a distance d apart, the focal length F of the system of lenses is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Combine the fractions in the right member.

90. In electricity, the true current reading I_t of a current meter is given by

$$I_t = I_r + \frac{R_m}{R_t} \cdot I_r$$

where I_r = the meter reading, R_m = the resistance of the current meter, and R_t = the total resistance of the circuit without the meter. Perform the indicated operations in the right member and write as a single expression in I_r, R_m, and R_t.

91. A lens maker's equation for making a lens is given by

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

where f is the focal length of the lens, n is the index of refraction, and R_1 and R_2 are the radii of curvature of the surfaces. Simplify the right member by performing the indicated operations and obtaining a single rational expression.

92. The expression

$$V_1\left(1 + \frac{T_2 - T_1}{T_1}\right) - V_2\left(\frac{T_2 - T_1}{T_2} - 1\right)$$

gives the volume change of a gas under constant pressure. Simplify the expression by performing the indicated operations.

Review exercises

Completely factor the following expressions. See section 3–8.

1. $8x^3 - 2x^2 + 12x - 3$
2. $4x^2 - 12x - 7$
3. $9y^2 - 49$
4. Simplify the expression $4[3 - 2(5 - 1) + 8]$. See section 1–4.
5. It costs Hank \$24 to rent a mower for one hour. Write a statement for the cost to rent the mower for k hours. See section 2–1.

4–4 ■ Complex rational expressions

The previous sections of this chapter have dealt with simple rational numbers and simple rational expressions—rational numbers and rational expressions that have a single integer, or single polynomial, in the numerator and the denominator. We now consider a **complex rational expression**—a rational expression whose numerator or denominator, or both, contain rational expressions. To illustrate,

$$\frac{\frac{2}{x}}{\frac{3}{x^2}}, \quad \frac{\frac{2}{x} - 3}{4 + \frac{1}{x}}, \quad \frac{\frac{5}{y} - \frac{3}{y}}{y - 3}, \quad \text{and} \quad \frac{\frac{p+1}{p-3}}{\frac{4-p}{p}}$$

are all complex rational expressions. To simplify such expressions, we eliminate the rational expression within the numerator and the denominator to obtain a simple rational expression.

Consider the complex rational expression

$$\frac{\left.\frac{p+1}{p-3}\right\} \text{Primary numerator}}{\left.\frac{4-p}{p}\right\} \text{Primary denominator}}$$

For the sake of discussion, we call

$$\frac{p+1}{\underbrace{p-3}_{\text{Secondary denominator}}}$$

the **primary numerator** and

$$\frac{4-p}{\underbrace{p}_{\text{Secondary denominator}}}$$

the **primary denominator.** The expressions $p - 3$ and p are called the **secondary denominators.** Thus, to simplify the complex rational expression, we must eliminate the secondary denominators. This can be accomplished in either one of the two ways.

Simplifying a complex rational expression

Method 1 Form a single rational expression in the primary numerator and the primary denominator and divide the primary numerator by the primary denominator.

Method 2 Multiply the primary numerator and the primary denominator by the LCM of the secondary denominators and reduce the resulting fraction to lowest terms.

Example 4–4 A

Simplify each complex rational expression using method 1. Assume all denominators are nonzero.

1. $\dfrac{\dfrac{2}{x}}{\dfrac{3}{x^2}} = \dfrac{2}{x} \div \dfrac{3}{x^2}$

$= \dfrac{2}{x} \cdot \dfrac{x^2}{3}$ Multiply by the reciprocal of $\dfrac{3}{x^2}$

$= \dfrac{2x^2}{3x}$ Multiply numerators and denominators

$= \dfrac{2x}{3}$ Reduce by x

2. $\dfrac{\dfrac{2}{x} - 3}{4 + \dfrac{1}{x}}$

$= \dfrac{\dfrac{2}{x} - \dfrac{3x}{x}}{\dfrac{4x}{x} + \dfrac{1}{x}} = \dfrac{\dfrac{2 - 3x}{x}}{\dfrac{4x + 1}{x}}$ Perform the indicated addition (in the denominator) and the indicated subtraction (in the numerator)

$= \dfrac{2 - 3x}{x} \cdot \dfrac{x}{4x + 1}$ Multiply by the reciprocal of $\dfrac{4x + 1}{x}$

$= \dfrac{x(2 - 3x)}{x(4x + 1)}$ Multiply numerators and denominators

$= \dfrac{2 - 3x}{4x + 1}$ Reduce by x

▶ ***Quick check*** Simplify $\dfrac{\dfrac{1}{x^2} - \dfrac{1}{y^2}}{\dfrac{1}{x} + \dfrac{1}{y}}$ using method 1. Assume all denominators are nonzero. ■

A second method to accomplish this same end is to find the least common multiple (LCM) of the *secondary denominators* and to apply the fundamental principle of rational expressions by multiplying the primary numerator and the primary denominator by the LCM, thus eliminating the secondary denominators.

Example 4–4 B

Simplify each complex rational expression using method 2. Assume all denominators are nonzero.

1. $\dfrac{\frac{7}{8}}{\frac{5}{6}}$

The LCM of the secondary denominators, 8 and 6, is 24.

$$\frac{\frac{7}{8}}{\frac{5}{6}} = \frac{\frac{7}{8}\cdot 24}{\frac{5}{6}\cdot 24}$$ Multiply the primary numerator and the primary denominator by 24

$$= \frac{7\cdot 3}{5\cdot 4}$$ Reduce in the numerator and the denominator

$$= \frac{21}{20}$$ Multiply in the numerator and the denominator

2. $\dfrac{\frac{p+1}{p-3}}{\frac{4-p}{p}}$

The LCM of the secondary denominators of $p - 3$ and p is $p\,(p - 3)$.

$$\frac{\frac{p+1}{p-3}}{\frac{4-p}{p}} = \frac{\frac{p+1}{p-3}\cdot p(p-3)}{\frac{4-p}{p}\cdot p(p-3)}$$ Multiply the primary numerator and the primary denominator by the LCM $p(p-3)$

$$= \frac{(p+1)p}{(4-p)(p-3)}$$ Reduce in numerator by $p - 3$ and in denominator by p

$$= \frac{p^2+p}{-p^2+7p-12}$$ Multiply in numerator and denominator

3. $\dfrac{\frac{5}{y}-\frac{3}{y}}{y-3}$

The LCM of the secondary denominators is y.

$$\frac{\frac{5}{y}-\frac{3}{y}}{y-3} = \frac{\left(\frac{5}{y}-\frac{3}{y}\right)\cdot y}{(y-3)\cdot y}$$ Multiply primary numerator and primary denominator by y

$$= \frac{\frac{5}{y}\cdot y-\frac{3}{y}\cdot y}{y(y-3)}$$ Apply distributive property

$$= \frac{5-3}{y(y-3)}$$ Reduce by y in the numerator

$$= \frac{2}{y(y-3)}$$ Subtract in the numerator

4. $$\frac{(y+2)+\dfrac{7}{y-6}}{(y-1)+\dfrac{4}{y-6}}$$

The LCM of the secondary denominators is $y-6$.

$$\frac{(y+2)+\dfrac{7}{y-6}}{(y-1)+\dfrac{4}{y-6}}=\frac{\left[(y+2)+\dfrac{7}{y-6}\right](y-6)}{\left[(y-1)+\dfrac{4}{y-6}\right](y-6)}$$

Multiply primary numerator and primary denominator by $y-6$

$$=\frac{(y+2)(y-6)+\dfrac{7}{y-6}\cdot(y-6)}{(y-1)(y-6)+\dfrac{4}{y-6}\cdot(y-6)}$$

Apply distributive property

$$=\frac{y^2-4y-12+7}{y^2-7y+6+4}$$

Perform indicated operations

$$=\frac{y^2-4y-5}{y^2-7y+10}$$

Combine like terms

$$=\frac{(y-5)(y+1)}{(y-5)(y-2)}$$

Factor numerator and denominator

$$=\frac{y+1}{y-2}$$

Reduce by $y-5$

5. $$\frac{y^{-1}-x^{-1}}{y^{-1}+x^{-1}}$$

By definition, $y^{-1}=\dfrac{1}{y}$ and $x^{-1}=\dfrac{1}{x}$. Thus,

$$\frac{y^{-1}-x^{-1}}{y^{-1}+x^{-1}}=\frac{\dfrac{1}{y}-\dfrac{1}{x}}{\dfrac{1}{y}+\dfrac{1}{x}}$$

The LCM of the secondary denominators is xy.

$$=\frac{\dfrac{1}{y}\cdot xy-\dfrac{1}{x}\cdot xy}{\dfrac{1}{y}\cdot xy+\dfrac{1}{x}\cdot xy}$$

Multiply the primary numerator and primary denominator by xy using the distributive property

$$=\frac{x-y}{x+y}$$

Reduce in each term

▶ ***Quick check*** Simplify $\dfrac{\dfrac{x-3}{x+2}}{\dfrac{x+1}{x-4}}$ using method 2. Assume all denominators are nonzero.

Note A reminder again—always check the factorability of the numerator and the denominator to *reduce* the simple rational expression to lowest terms.

Mastery points

Can you

- Recognize a complex rational expression?
- Simplify a complex rational expression?

Exercise 4–4

Simplify each complex rational expression using method 1. Assume all denominators are nonzero. See example 4–4 A.

Example $\dfrac{\dfrac{1}{x^2} - \dfrac{1}{y^2}}{\dfrac{1}{x} + \dfrac{1}{y}}$

Solution $= \dfrac{\dfrac{y^2 - x^2}{x^2y^2}}{\dfrac{y + x}{xy}}$ Subtract in the numerator and add in the denominator

$= \dfrac{y^2 - x^2}{x^2y^2} \cdot \dfrac{xy}{y + x}$ Multiply by the reciprocal of $\dfrac{y + x}{xy}$

$= \dfrac{(y - x)(y + x)xy}{x^2y^2(y + x)}$ Factor and multiply numerators and denominators

$= \dfrac{y - x}{xy}$ Reduce by common factors xy and $y + x$

1. $\dfrac{\frac{3}{5}}{\frac{4}{7}}$

2. $\dfrac{\frac{5}{6}}{\frac{7}{8}}$

3. $\dfrac{\frac{2}{3} + \frac{1}{4}}{\frac{1}{2} - \frac{1}{4}}$

4. $\dfrac{\frac{1}{3} + \frac{1}{2}}{\frac{2}{3} - \frac{5}{6}}$

5. $\dfrac{4 - \frac{2}{5}}{3 + \frac{3}{10}}$

6. $\dfrac{\frac{4}{x}}{\frac{8}{x^2}}$

7. $\dfrac{\frac{7}{m^2}}{\frac{8}{m}}$

8. $\dfrac{\frac{5}{b}}{\frac{9}{b - 3}}$

9. $\dfrac{\frac{-4}{y + 1}}{\frac{3}{y}}$

10. $\dfrac{\frac{8}{a - 3}}{\frac{-6}{a + 2}}$

11. $\dfrac{\frac{x}{x + 7}}{\frac{x}{x - 3}}$

12. $\dfrac{\frac{m - 3}{4}}{\frac{2m + 7}{6}}$

13. $\dfrac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a^2} - \frac{1}{b^2}}$

14. $\dfrac{x - y}{\frac{1}{x} - \frac{1}{y}}$

15. $\dfrac{\frac{x}{y} + 2}{\frac{x}{y} - \frac{4y}{x}}$

16. $\dfrac{\frac{2}{q} - \frac{3}{r}}{2 + \frac{1}{qr}}$

Simplify each complex rational expression by multiplying the numerator and the denominator by the LCM of the secondary denominators. See example 4–4 B.

Example $\dfrac{\dfrac{x-3}{x+2}}{\dfrac{x+1}{x-4}}$

Solution

$$= \frac{\dfrac{x-3}{x+2}\cdot(x+2)(x-4)}{\dfrac{x+1}{x-4}\cdot(x+2)(x-4)}$$

Multiply primary numerator and primary denominator by LCM $(x+2)(x-4)$

$$= \frac{(x-3)(x-4)}{(x+1)(x+2)}$$

Reduce by common factors

$$= \frac{x^2-7x+12}{x^2+3x+2}$$

Perform indicated multiplication

17. $\dfrac{\frac{5}{9}}{\frac{3}{4}}$

18. $\dfrac{\frac{1}{6}-\frac{7}{8}}{\frac{2}{3}+\frac{1}{4}}$

19. $\dfrac{\frac{5}{y^2}}{\frac{7}{y}}$

20. $\dfrac{\frac{3}{x-2}}{\frac{9}{x}}$

21. $\dfrac{\frac{7}{a+9}}{\frac{-4}{a-5}}$

22. $\dfrac{\frac{n-9}{n+2}}{\frac{n}{3n-5}}$

23. $\dfrac{\frac{4x^2-y^2}{3x}}{\frac{2x+y}{4x}}$

24. $\dfrac{\frac{x+3}{y}}{\frac{x^2-9y^2}{6y}}$

25. $\dfrac{4-\frac{3}{x+3}}{5+\frac{6}{x-1}}$

26. $\dfrac{\frac{7}{y-3}+8}{9-\frac{1}{2y+3}}$

27. $\dfrac{\frac{1}{x}+\frac{1}{y^2}}{\frac{1}{x^2}-\frac{1}{x}}$

28. $\dfrac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{a^2}+\frac{1}{b^2}}$

29. $\dfrac{m+n}{\frac{1}{m}-\frac{1}{n}}$

30. $\dfrac{x-y}{\frac{1}{x}+\frac{1}{y}}$

31. $\dfrac{\frac{5}{p^2}+\frac{4}{q}}{p-q}$

32. $\dfrac{\frac{1}{m^2}-\frac{1}{n^2}}{m+n}$

33. $\dfrac{(a+5)+\frac{3}{a+4}}{(a+3)-\frac{5}{a+4}}$

34. $\dfrac{(x-3)+\frac{7}{2x+1}}{(x+9)-\frac{3}{2x+1}}$

35. $\dfrac{y-\frac{3}{4y-3}}{(y+2)+\frac{3}{y+5}}$

36. $\dfrac{(m-3)+\frac{6}{2m+3}}{m-\frac{9}{m-6}}$

37. $\dfrac{\frac{t^2-2t-8}{t^2+7t+6}}{\frac{t^2-t-6}{t^2+2t+1}}$

38. $\dfrac{\frac{y^2-5y-14}{y^2+3y-10}}{\frac{y^2-8y+7}{y^2+6y+5}}$

39. $\dfrac{\frac{3}{x^2-x-6}}{\frac{2}{x+2}-\frac{4}{x-3}}$

40. $\dfrac{\frac{9}{a-7}+\frac{8}{2a+3}}{\frac{10}{2a^2-11a-21}}$

41. $\dfrac{5+\frac{4}{b-1}}{\frac{7}{b+5}-\frac{3}{b-1}}$

42. $\dfrac{\frac{6}{x+5}-7}{\frac{8}{x+5}-\frac{9}{x+3}}$

43. $\dfrac{\frac{1}{1-x}-\frac{1}{1+x}}{\frac{1+x}{1-x}-\frac{1-x}{1+x}}$

44. $\dfrac{\dfrac{a-1}{a-b}-\dfrac{a-b}{a-1}}{\dfrac{1}{a-1}-\dfrac{1}{a-b}}$

45. $\dfrac{\dfrac{3}{ab}+\dfrac{4}{bc}-\dfrac{2}{ac}}{\dfrac{5}{abc}}$

46. $\dfrac{\dfrac{x}{yz}-\dfrac{y}{xz}+\dfrac{z}{xy}}{\dfrac{1}{x^2y^2}-\dfrac{1}{x^2z^2}+\dfrac{1}{y^2z^2}}$

47. $\dfrac{x^{-1}+y^{-1}}{x^{-1}-y^{-1}}$

48. $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}}$

49. $\dfrac{x^{-1}}{x^{-1}+y^{-1}}$

50. $\dfrac{x^{-2}}{x^{-2}-y^{-2}}$

51. $\dfrac{p^{-2}+q^{-2}}{q^{-2}}$

52. $\dfrac{2x^{-1}-y^{-1}}{x^{-1}+5y^{-1}}$

53. $(p^{-1}-q^{-1})^{-1}$

54. $(x^{-1}+y^{-1})^{-1}$

Solve the following word problems.

55. In electricity, a relationship for the current, i, in a capacitor of "size" C is given by

$$i=\frac{V_s-\dfrac{it}{C}}{R}$$

Simplify the right member.

56. In electricity, the voltage between two adjacent nodes, denoted by $V_{AA'}$, is given by Millman's Theorem,

$$V_{AA'}=\frac{\dfrac{V_{S1}}{R_1}+\dfrac{V_{S2}}{R_2}+\dfrac{V_{S3}}{R_3}}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}}$$

where V_{S1}, V_{S2}, V_{S3} represent equivalent voltages of the branches between nodes A and A', whereas R_1, R_2, and R_3 are equivalent resistances of the branches between nodes A and A'. Simplify the right member.

57. A refrigeration coefficient, CP, of performance formula for the ideal refrigerator is given by

$$CP=\frac{1}{\dfrac{T_2}{T_1}-1}$$

Simplify the right member.

58. The capacitance C of a circuit connecting three capacitances C_1, C_2, and C_3 in series is given by

$$C=\frac{1}{\dfrac{1}{C_1}+\dfrac{1}{C_2}+\dfrac{1}{C_3}}$$

Simplify the right member.

59. When making a round trip whose one-way distance is d, the average rate (speed) traveled, r, is given by

$$r=\frac{2d}{\dfrac{d}{r_1}+\dfrac{d}{r_2}}$$

where r_1 is the average rate going and r_2 is the average rate coming back. Simplify the right member.

Review exercises

Find the solution set of the following equations. See sections 2–1 and 2–4.

1. $\dfrac{1}{2}x+3=\dfrac{2}{3}$

2. $|x-3|=5$

3. $5(2x-4)+3x=2x+2$

Find the solution set of the following inequalities. See section 2–6.

4. $|x-2|<4$

5. $|2x+5|\geq 3$

6. Perform the indicated subtraction. See section 3–2.
$(3a^3+2a^2-3)-(a^3-6a^2+a+1)$

4–5 ■ Quotients of polynomials

Division of a polynomial by a monomial

The first type of division that we will study is that of a polynomial divided by a monomial. Consider the indicated division

$$\frac{4a^3 - 12a^2 + 8a}{2a}$$

Using the meaning of division, we can rewrite this as

$$(4a^3 - 12a^2 + 8a) \cdot \frac{1}{2a}$$

and applying the distributive property, we have

$$\frac{4a^3 - 12a^2 + 8a}{2a} = \frac{4a^3}{2a} - \frac{12a^2}{2a} + \frac{8a}{2a}$$
$$= 2a^2 - 6a + 4$$

Dividing a polynomial by a monomial

For monomials $a_1, a_2, \cdots, a_n$, and d, where $d \neq 0$,

$$\frac{a_1 + a_2 + \cdots + a_n}{d} = \frac{a_1}{d} + \frac{a_2}{d} + \cdots + \frac{a_n}{d}$$

Concept

To divide a polynomial by a monomial, simply divide each term of the polynomial by the monomial and write the resulting quotients.

■ **Example 4–5 A**

Perform the indicated division. Assume that no denominator equals zero.

1. $\dfrac{4a^6 + 16a^4 - 6a}{4a^2} = \dfrac{4a^6}{4a^2} + \dfrac{16a^4}{4a^2} - \dfrac{6a}{4a^2}$ Divide each term by monomial $4a^2$

$= a^4 + 4a^2 - \dfrac{3}{2a}$ Reduce to lowest terms

2. $\dfrac{x(3y-2) + 2(3y-2)}{3y-2} = \dfrac{x(3y-2)}{(3y-2)} + \dfrac{2(3y-2)}{(3y-2)}$ Divide each term by $(3y - 2)$

$= x + 2$ Reduce to lowest terms

Note A common error in this type of problem is demonstrated in the example

$$\frac{x^3 + x^2}{x^2} = \frac{x^3 + \cancel{x^2}^{\,1}}{\cancel{x^2}_{\,1}} = \frac{x^3 + 1}{1} = x^3 + 1$$

It is tempting to "cancel" the x^2 in the numerator with the x^2 in the denominator, but the correct procedure is

$$\frac{x^3 + x^2}{x^2} = \frac{x^3}{x^2} + \frac{x^2}{x^2} = x + 1$$

▶ ***Quick check*** Simplify $\dfrac{5x^3 - 10x^2 + 25x}{5x^2}$. Assume the denominator is nonzero. ■

Division of a polynomial by a polynomial

The second type of division that we will study is that of a polynomial divided by another polynomial. Consider the example

$$\frac{2x^2 - x - 15}{x - 3}$$

The example is set up so that the divisor, $x - 3$, and the dividend, $2x^2 - x - 15$, are arranged in descending powers of the variable.

$$x - 3\overline{)2x^2 - x - 15}$$

Now we divide the first term of the dividend, $2x^2$, by the first term of the divisor, x. The result is $\dfrac{2x^2}{x} = 2x$, and we place the $2x$ above the division line.

$$\begin{array}{r} 2x \\ x - 3\overline{)2x^2 - x - 15} \end{array}$$

Next, we multiply $x - 3$ and $2x$, placing the product below the dividend, and subtract.

$$\begin{array}{rl} 2x & \\ x - 3\overline{)2x^2 - x - 15} & \\ \underline{2x^2 - 6x} & \qquad 2x(x - 3) = 2x^2 - 6x \end{array}$$

Recall that when we subtract, we change the signs and add. Then $(2x^2 - x) - (2x^2 - 6x) = 2x^2 - x - 2x^2 + 6x = 5x$.

$$\begin{array}{r} 2x \\ x - 3\overline{)2x^2 - x - 15} \\ -\underline{2x^2 - 6x} \\ 5x \end{array}$$

Now we bring down the -15 and repeat the same process.

$$\begin{array}{r} 2x \\ x - 3\overline{)2x^2 - x - 15} \\ \underline{2x^2 - 6x} \\ 5x - 15 \end{array}$$

Divide the $5x$ by x, which results in 5, and multiply 5 and $x - 3$.

$$\begin{array}{r} 2x + 5 \\ x - 3\overline{)2x^2 - x - 15} \\ \underline{2x^2 - 6x} \\ 5x - 15 \\ \underline{5x - 15} \end{array}$$

Subtract $(5x - 15) - (5x - 15) = 5x - 15 - 5x + 15 = 0$.

$$\begin{array}{r} 2x + 5 \\ x - 3\overline{)2x^2 - x - 15} \\ \underline{2x^2 - 6x} \\ 5x - 15 \\ \underline{5x - 15} \\ 0 \end{array}$$

We subtract and get a remainder of zero, so the quotient is $2x + 5$.

$$\frac{2x^2 - x - 15}{x - 3} = 2x + 5$$

To check the problem, multiply the quotient times the divisor to get the dividend.

$$(2x + 5)(x - 3) = 2x^2 - x - 15$$

Note *It is important* when dividing a polynomial by a polynomial that both the divisor and the dividend have their terms arranged in descending powers of the variable.

■ Example 4–5 B

Find the indicated quotients. Assume no divisor is equal to zero.

1. $(2x^2 - 11x + 15) \div (x - 3)$

$$\begin{array}{r|l}
x - 3 & \overline{2x - 5} \\
 & 2x^2 - 11x + 15 \\
 & \underline{2x^2 - 6x} \qquad \text{Subtract (change signs and add)} \\
 & -5x + 15 \\
 & \underline{-5x + 15} \qquad \text{Subtract (change signs and add)} \\
 & 0
\end{array}$$

Therefore $\dfrac{2x^2 - 11x + 15}{x - 3} = 2x - 5$.

2. $(2y^3 - y^2 + 5y + 5) \div (2y + 1)$

$$\begin{array}{r|l}
2y + 1 & \overline{y^2 - y + 3} \\
 & 2y^3 - y^2 + 5y + 5 \\
 & \underline{2y^3 + y^2} \qquad \text{Subtract (change signs and add)} \\
 & -2y^2 + 5y \\
 & \underline{-2y^2 - y} \qquad \text{Subtract (change signs and add)} \\
 & 6y + 5 \\
 & \underline{6y + 3} \qquad \text{Subtract (change signs and add)} \\
 & 2 \qquad \text{Remainder}
\end{array}$$

When there is a remainder, as in this example, we write the remainder over the divisor.

$$\frac{2y^3 - y^2 + 5y + 5}{2y + 1} = y^2 - y + 3 + \frac{2}{2y + 1}$$

3. $(2x^3 - x + 5) \div (x - 1)$

There is no term in the dividend that contains x^2. Therefore we will insert $0x^2$ as a placeholder so that all powers of the variable x are present in descending order. The value of the dividend has not changed since we added $0x^2$, which is another name for 0.

$$\begin{array}{r|l}
x - 1 & \overline{2x^2 + 2x + 1} \\
 & 2x^3 + 0x^2 - x + 5 \\
 & \underline{2x^3 - 2x^2} \qquad \text{Subtract (change signs and add)} \\
 & 2x^2 - x \\
 & \underline{2x^2 - 2x} \qquad \text{Subtract (change signs and add)} \\
 & x + 5 \\
 & \underline{x - 1} \qquad \text{Subtract (change signs and add)} \\
 & 6 \qquad \text{Remainder}
\end{array}$$

Hence $\dfrac{2x^3 - x + 5}{x - 1} = 2x^2 + 2x + 1 + \dfrac{6}{x - 1}$.

▶ ***Quick check*** Divide $\dfrac{3y^3 + 5y^2 - 11y + 6}{3y - 1}$. Assume the denominator is nonzero. ■

Mastery points

Can you

- Divide a polynomial by a monomial?
- Divide a polynomial by a polynomial?

Exercise 4–5

Perform the indicated divisions. Assume that no divisor is equal to zero. See example 4–5 A.

Example $\dfrac{5x^3 - 10x^2 + 25x}{5x^2}$

Solution $= \dfrac{5x^3}{5x^2} - \dfrac{10x^2}{5x^2} + \dfrac{25x}{5x^2}$ Divide denominator into each term

$= x - 2 + \dfrac{5}{x}$ Divide in each term by common factors

1. $\dfrac{25x^2 - 15x + 10}{5}$
2. $\dfrac{2a^4 - 3a^2 + a}{a}$
3. $\dfrac{4x^4 - 8x^3 + 12x}{-4x}$
4. $\dfrac{15y^5 + 25y^2 + 10y}{-5y^2}$
5. $\dfrac{ac^2 - ac}{ac}$
6. $\dfrac{bx - b^2x^2}{bx}$
7. $\dfrac{30x^3y^4 + 21x^2y^2 - 18x^2y^4}{3x^2y^2}$
8. $\dfrac{36x^4y^2z^3 - 24x^2y^5z + 18x^2y^2z^2}{6x^2y^3z}$
9. $\dfrac{21a^7b^2c^3 - 35a^5b^5c^3 + 49a^4b^2c^3}{7abc^3}$
10. $\dfrac{x(y - 2) - z(y - 2)}{y - 2}$
11. $\dfrac{2a(b - 4) + 3c(b - 4)}{b - 4}$
12. $\dfrac{x^2y(z + 3) - 3x^4y^3(z + 3)}{xy(z + 3)}$

See example 4–5 B.

Example $\dfrac{3y^3 + 5y^2 - 11y + 6}{3y - 1}$

Solution

$$\begin{array}{r l}
 & y^2 + 2y - 3 \\
3y - 1 \,\big) & \overline{3y^3 + 5y^2 - 11y + 6} \\
 & \underline{3y^3 - y^2} \qquad \text{Subtract (change signs and add)} \\
 & 6y^2 - 11y \\
 & \underline{6y^2 - 2y} \qquad \text{Subtract (change signs and add)} \\
 & -9y + 6 \\
 & \underline{-9y + 3} \qquad \text{Subtract (change signs and add)} \\
 & 3 \qquad \text{Remainder}
\end{array}$$

$\dfrac{3y^3 + 5y^2 - 11y + 6}{3y - 1} = y^2 + 2y - 3 + \dfrac{3}{3y - 1}$

13. $(a^2 - 2a - 8) \div (a + 2)$

14. $(y^2 + 2y - 3) \div (y + 3)$

15. $(y^2 + 7y + 11) \div (y + 5)$

$\boxed{16.}$ $(2x - 5 + x^2) \div (x + 4)$

17. $(10 - 7x + x^2) \div (x - 3)$

18. $\dfrac{a^2 - 5a + 1}{a - 1}$

19. $\dfrac{a^3 + a^2 - 2a + 12}{a + 3}$

20. $\dfrac{y^3 + 3y^2 - y - 6}{y + 2}$

21. $\dfrac{2y^3 - y^2 - 2y + 3}{y + 1}$

22. $\dfrac{3x^3 - 4x^2 - 5x - 4}{x + 2}$

23. $\dfrac{5x^2 - 11x + 2x^3 + 4}{2x - 1}$

24. $\dfrac{3a^3 - 2 - 5a - a^2}{3a + 2}$

25. $\dfrac{4a^3 + 8a^2 - 5a + 1}{2a + 1}$

26. $\dfrac{6y^3 - y^2 - 11y + 10}{3y - 2}$

$\boxed{27.}$ $\dfrac{9y^3 + 11y + 6}{3y + 1}$

28. $\dfrac{27x^3 - 1}{3x - 1}$

29. $\dfrac{a^3 + 27}{a + 3}$

30. $\dfrac{x^4 - 16}{x - 2}$

31. $\dfrac{3x^4 - 2x^3 + x - 1}{x + 1}$

$\boxed{32.}$ $\dfrac{2x^3 + 5x^2 + 5x + 3}{x^2 + x + 1}$

33. $\dfrac{3a^3 - 4a^2 + 10a - 3}{a^2 - a + 3}$

34. $\dfrac{a^4 - 2a^2 - 3a - 1}{a^2 - 2a - 1}$

35. $\dfrac{y^4 + 4y^3 + 3y^2 - 2y - 1}{y^2 + 3y + 1}$

36. $\dfrac{y^4 - y^3 - 11y^2 + 10y + 2}{y^2 - 4y + 2}$

37. $\dfrac{x^4 + 4x^3 + x^2 - 10x - 12}{x^2 + x - 4}$

$\boxed{38.}$ $\dfrac{2x^4 - x^3 + 5x^2 - x + 3}{x^2 + 1}$

39. $\dfrac{3a^4 - 2a^3 + 2a^2 - 4a - 8}{a^2 + 2}$

40. $\dfrac{a^4 + 4a^3 + a^2 - 10a - 9}{a^2 + 3a + 2}$

41. $\dfrac{2y^4 - 3y^3 + 8y^2 - 9y + 8}{2y^2 - 3y + 2}$

42. $\dfrac{3y^4 + y^3 - 8y^2 - 3y - 5}{3y^2 + y + 1}$

Solve the following word problems.

43. Evaluate $2y^3 - y^2 - 2y + 3$ when $y = -1$ and compare this answer to the remainder found in exercise 21.

44. Evaluate $3x^3 - 4x^2 - 5x - 4$ when $x = -2$ and compare this answer to the remainder found in exercise 22.

45. The area of a rectangle is found by multiplying the length times the width. If the area of a rectangle is $6x^2 - 17x + 12$ and the length is $3x - 4$, find the width.

46. A contractor uses the expression $x^2 + 6x + 8$ to represent the square footage of a room. If she decides that the length of the room will be represented by $x + 4$, what will the width of the room be in terms of x?

47. The volume of a box is found by multiplying the length times the width times the height. If the volume of a box is $6x^3 + 11x^2 - 19x + 6$, the height is $x + 3$, and the width is $2x - 1$, find the length.

48. An electrician uses the expression $x^2 + 5x + 6$ to determine the amount of wire to order when wiring a house. If the formula comes from multiplying the number of rooms times the number of outlets and he knows the number of rooms to be $x + 2$, find the number of outlets in terms of x.

textbook revolution
TAKING THE BITE OUT OF BOOKS

Review exercises

Simplify the following expressions. Assume all denominators are nonzero. Answer with positive exponents only. See sections 3–1 and 3–3.

1. $\dfrac{x^3y}{x^2y}$

2. $\dfrac{x^{-3}}{x^{-6}}$

3. $(2a^{-4}b^0)^3$

4. Simplify the expression $\dfrac{\frac{3}{4}-\frac{1}{2}}{\frac{2}{3}+\frac{1}{6}}$ See section 4–4.

Perform the indicated operations. Assume all denominators are nonzero. See sections 4–2 and 4–3.

5. $\dfrac{2x-1}{x^2-9}-\dfrac{x+1}{x+3}$

6. $\dfrac{2x-1}{3x-1}\div\dfrac{4x}{6x-2}$

7. Find the solution set of the equation $\dfrac{1}{5}x-\dfrac{2}{3}=\dfrac{3}{5}$. See section 2–1.

4–6 ■ Synthetic division, the remainder theorem, and the factor theorem

In section 4–5, we studied the procedure for dividing a polynomial by another polynomial. Many times the divisor is a binomial of the form $x-k$, k is a constant. We can shorten the process considerably by using **synthetic division.**

Consider the following example: $(2x^3+5x^2+x-1)\div(x+2)$. Performing the indicated division, we obtain

$$
\begin{array}{lrl}
 & 2x^2 + x - 1 & \text{Quotient} \\
\text{Divisor} \quad x+2 & \overline{)\,2x^3+5x^2+x-1} & \text{Dividend} \\
 & \underline{2x^3+4x^2} & \\
 & x^2+x & \\
 & \underline{x^2+2x} & \\
 & -x-1 & \\
 & \underline{-x-2} & \\
 & 1 & \text{Remainder}
\end{array}
$$

We can write the exact same problem using only the coefficients of the terms.

$$
\begin{array}{l|rrrr|l}
 & 2 & 1 & -1 & & \text{Coefficients from the quotient} \\
\text{Coefficients from the divisor} \rightarrow 1 \quad 2 & 2 & 5 & 1 & -1 & \text{Coefficients from the dividend} \\
 & \underline{2} & \underline{4} & & & \\
 & & 1 & 1 & & \\
 & & \underline{1} & \underline{2} & & \\
 & & & -1 & -1 & \\
 & & & \underline{-1} & \underline{-2} & \\
 & & & & 1 & \text{Remainder}
\end{array}
$$

Observe that the circled numbers are repetitions of the numbers directly above them. We can rewrite the problem without them.

$$
\begin{array}{cc|rrrr}
 & & 2 & 1 & -1 & \\
\hline
1 & 2 & 2 & 5 & 1 & -1 \\
 & & & 4 & & \\
\cline{4-5}
 & & & 1 & 1 & \\
 & & & & 2 & \\
\cline{4-6}
 & & & & -1 & -1 \\
 & & & & & -2 \\
\cline{6-6}
 & & & & & 1
\end{array}
$$

The circled numbers are again the same as the numbers directly above them. Therefore we can omit them.

$$
\begin{array}{cc|rrrr}
 & & 2 & 1 & -1 & \\
\hline
1 & 2 & 2 & 5 & 1 & -1 \\
 & & & 4 & & \\
\cline{4-4}
 & & & 1 & & \\
 & & & & 2 & \\
\cline{5-5}
 & & & & -1 & \\
 & & & & & -2 \\
\cline{6-6}
 & & & & & 1
\end{array}
$$

All of the numbers can be condensed. We omit the top row of numbers since it duplicates the bottom set of numbers. We shall also omit the 1 at the upper left.

$$
\begin{array}{r|rrrr}
2 & 2 & 5 & 1 & -1 \\
 & & 4 & 2 & -2 \\
\hline
 & 2 & 1 & -1 & 1
\end{array}
$$

By changing the 2 at the upper left to its additive inverse, -2, and adding the additive inverse in each step, instead of subtracting, we obtain the same result. The following is the final form to be used when performing synthetic division.

$$
\begin{array}{r|rrrr}
-2 & 2 & 5 & 1 & -1 \\
 & & -4 & -2 & 2 \\
\hline
 & 2 & 1 & -1 & 1 \\
 & \uparrow & \uparrow & \uparrow & \nwarrow \\
 & x^2 & x & \text{Constant} & \text{Remainder}
\end{array}
$$

The bottom line of this synthetic division process represents the coefficients of the terms of the quotient along with the remainder. That is, 2 1 -1 1 represents the quotient $2x^2 + x - 1$ with a remainder of 1.

Note The degree 2 of the quotient is one less than the degree 3 of the dividend. This will *always* be the case.

Example 4–6 A

Divide the following using synthetic division. Assume no denominator equals zero.

1. $(3x^2 + 7x + 6) \div (x + 1)$
To use synthetic division, the divisor *must* be of the form $x - k$. Therefore we write $x + 1$ as $x - (-1)$ so $k = -1$. Now write the coefficients of the terms in $3x^2 + 7x + 6$ with -1 to the left.

$$\begin{array}{r|rrr} -1 & 3 & 7 & 6 \\ & & & \\ \hline & & & \end{array}$$

First, we bring down 3 and multiply that by -1. This product, -3, is added to 7.

$$\begin{array}{r|rrr} -1 & 3 & 7 & 6 \\ & \downarrow & -3 & \\ \hline & 3 & 4 & \end{array}$$

The 4 is now multiplied by -1 and the product, -4, is added to 6.

$$\begin{array}{r|rrr} -1 & 3 & 7 & 6 \\ & & -3 & -4 \\ \hline & 3 & 4 & 2 \end{array}$$

We can now read the coefficients of the quotient and the remainder from the last row. The answer is $3x + 4 + \dfrac{2}{x+1}$. (2 is the remainder.)

2. $(2x^3 - 7x^2 - x + 12) \div (x - 3)$
The divisor, $x - 3$, is already in the form $x - k$, where $k = 3$. We set up the problem as follows:

$$\begin{array}{r|rrrr} 3 & 2 & -7 & -1 & 12 \\ & & & & \\ \hline & & & & \end{array}$$

We bring down 2 and begin the repetitive process of multiplying the last number in the bottom row times k, that is, 3, and adding this to the value in the next column.

$$\begin{array}{r|rrrr} 3 & 2 & -7 & -1 & 12 \\ & \downarrow & 6 & -3 & -12 \\ \hline & 2 & -1 & -4 & 0 \end{array}$$

Since we have a zero in the last position of the bottom row, there is no remainder and the quotient is $2x^2 - x - 4$.

3. $(x^3 - 11x + 8) \div (x - 3)$
Since the x^2 term is missing in the dividend, we think of the expression as $x^3 + 0x^2 - 11x + 8$ when writing down the coefficients.

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -11 & 8 \\ & \downarrow & 3 & 9 & -6 \\ \hline & 1 & 3 & -2 & 2 \end{array}$$

The answer is then $x^2 + 3x - 2 + \dfrac{2}{x-3}$.

4. $(2x^4 - 10x^3 + 11x^2 - 11x + 8) \div (x - 2)$

$$\begin{array}{r|rrrrr} 2 & 2 & -10 & 11 & -11 & 8 \\ & \downarrow & 4 & -12 & -2 & -26 \\ \hline & 2 & -6 & -1 & -13 & -18 \end{array}$$

The answer is $2x^3 - 6x^2 - x - 13 - \dfrac{18}{x-2}$.

▶ ***Quick check*** Use synthetic division to divide $(x^3 + 10x - 2) \div (x + 2)$. ■

Remainder and factor theorems

From our work with synthetic division, we can see that division by a polynomial in x, $P(x)$, by a polynomial $x - r$ results in a quotient $Q(x)$ and a constant remainder R. That is,

$$\frac{P(x)}{x-r} = Q(x) + R$$

and so

$$P(x) = (x - r)Q(x) + R$$

Using this equation, we evaluate $P(r)$ when $x = r$.

$$\begin{aligned} P(x) &= (x - r)Q(x) + R \\ P(r) &= (r - r)Q(r) + R \\ &= 0 \cdot Q(r) + R \\ &= 0 + R \\ &= R \end{aligned}$$

Thus, $P(r) = R$ and we find that the value of the polynomial when $x = r$ is the remainder R. We have just proved the **Remainder Theorem.**

Remainder theorem

If a polynomial $P(x)$ is divided by $x - r$, where r is a real number, the remainder R is $P(r)$.

■ **Example 4–6 B**

1. Determine the remainder when $P(x) = 2x^3 - 5x^2 + 4x - 2$ is divided by $x - 3$. Evaluate $P(3)$ using substitution.
   a. Using synthetic division,

$$\begin{array}{r|rrrr} 3 & 2 & -5 & 4 & -2 \\ & \downarrow & 6 & 3 & 21 \\ \hline & 2 & 1 & 7 & 19 \end{array} \longleftarrow \text{Remainder}$$

   Thus, $P(3) = 19$

b. Using substitution, we must show that $P(3) = 19$.

$$\begin{aligned} P(x) &= 2x^3 - 5x^2 + 4x - 2 \\ P(3) &= 2(3)^3 - 5(3)^2 + 4(3) - 2 \\ &= 54 - 45 + 12 - 2 \\ &= 19 \end{aligned}$$

Thus, we have shown that $R = 19$ is the result in either case.

▶ ***Quick check*** Using synthetic division and the remainder theorem, find the remainder when $P(x) = 3x^3 - 2x^2 + 4x - 5$ is divided by $x + 4$. Evaluate $P(-4)$ using substitution. ■

The **factor theorem**, which is a direct result of the remainder theorem (a corollary), applies when the remainder $R = 0$. This theorem shows there is a close relationship between the *factors* of a polynomial $P(x)$ and the values of x for which $P(x) = 0$.

Factor theorem

Given polynomial $P(x)$ and real number r, then

$x - r$ is a factor of $P(x)$ if and only if $P(r) = 0$

■ ***Example 4–6 C***

1. Show that $x + 2$ is a factor of $P(x) = x^3 - 3x^2 - 6x + 8$.
By the factor theorem, $x + 2$ is a factor of $P(x)$ if $P(-2) = 0$. Using synthetic division, we show that the remainder $R = 0$.

$$\begin{array}{r|rrrr} -2 & 1 & -3 & -6 & 8 \\ & \downarrow & -2 & 10 & -8 \\ \hline & 1 & -5 & 4 & \boxed{0} \end{array}$$

Thus, by the remainder theorem, $P(-2) = 0$ and by the factor theorem $x + 2$ is a factor of $P(x)$.

Note Alternatively, we can evaluate

$$\begin{aligned} P(-2) &= (-2)^3 - 3(-2)^2 - 6(-2) + 8 \\ &= -8 - 12 + 12 + 8 \\ &= 0 \end{aligned}$$

2. Find the solution set of the equation $x^3 - 3x^2 - 6x + 8 = 0$.
In example 1, we determined that $x + 2$ is a factor of $x^3 - 3x^2 - 6x + 8$ and the quotient $\dfrac{P(x)}{x + 2} = x^2 - 5x + 4$.
Thus, we have

$$\begin{aligned} (x + 2)(x^2 - 5x + 4) &= 0 \\ (x + 2)(x - 4)(x - 1) &= 0 \quad \text{Factor the left member} \end{aligned}$$

Since $x = 4$ when $x - 4 = 0$ and $x = 1$ when $x - 1 = 0$, the solution set is $\{-2, 4, 1\}$.

Note We have used the zero product property stated in section 4–1 to find the remaining solutions.

3. Find a polynomial $P(x)$ of degree 3 whose zeros are 2, -1, and -3.
   By the factor theorem, $(x - 2)$, $(x + 1)$, and $(x + 3)$ are the factors of $P(x)$. Thus,

   $$P(x) = (x - 2)(x + 1)(x + 3) = x^3 + 2x^2 - 5x - 6$$

   is a polynomial of degree 3 having the given zeros.

Note This polynomial is *not unique*, since multiplying $P(x)$ by any nonzero real number will yield another polynomial having the same zeros. To illustrate,

$$P(x) = 4x^3 + 8x^2 - 20x - 24$$

also has zeros 2, -1, and -3.

▶ ***Quick check*** Show that $x + 5$ is a factor of $P(x) = x^3 + 4x^2 - 7x - 10$. Find the solution set of the equation $x^3 + 4x^2 - 7x - 10 = 0$. ■

Given

$$P(x) = 4x^2 - 12x + 9$$

factoring the polynomial, $P(x) = (2x - 3)(2x - 3)$ and the zeros are $\frac{3}{2}$ and $\frac{3}{2}$. Thus, we may have repeated zeros. In this case, $\frac{3}{2}$ is repeated twice and we say the $P(x)$ has zero $\frac{3}{2}$ of *multiplicity 2*. In general, if a factor $x - r$ is repeated k times in a polynomial, we say r is a *zero of multiplicity k*.

■ Example 4–6 D

1. Given $P(x) = (x + 2)^2(x - 5)^3(x - 1)^4$, the distinct zeros are -2, 5, and 1. Since each is repeated more than once, we say -2 is a zero of multiplicity 2, 5 is a zero of multiplicity 3, and 1 is a zero of multiplicity 4.

2. Given 3 is a zero of the polynomial $P(x) = 2x^3 - 7x^2 - 7x + 30$, find the other zeros.
   Since 3 is a zero, then $x - 3$ is a factor of $P(x)$ by the factor theorem. Using synthetic division,

   $$\begin{array}{r|rrrr} 3 & 2 & -7 & -7 & 30 \\ & \downarrow & 6 & -3 & -30 \\ \hline & 2 & -1 & -10 & 0 \end{array}$$

   Thus, $P(x) = (x - 3)(2x^2 - x - 10)$ and factoring $2x^2 - x - 10$,

   $$P(x) = (x - 3)(2x - 5)(x + 2)$$

   Since $x = 5/2$ when $2x - 5 = 0$ and $x = -2$ when $x + 2 = 0$, the zeros of $P(x)$ are 3, $5/2$, and -2.

3. Given -1 is a solution (root) of the equation

$$3x^4 - 7x^3 - 33x^2 - 33x - 10 = 0$$

of multiplicity 2, find the solution set.
Given -1 is a solution of multiplicity 2, then $(x + 1)^2 = x^2 + 2x + 1$ is a factor of the left member of the equation. We divide

$$\begin{array}{r} 3x^2 - 13x - 10 \\ x^2 + 2x + 1 \overline{) 3x^4 - 7x^3 - 33x^2 - 33x - 10} \\ \underline{3x^4 + 6x^3 + 3x^2} \\ -13x^3 - 36x^2 - 33x \\ \underline{-13x^3 - 26x^2 - 13x} \\ -10x^2 - 20x - 10 \\ \underline{-10x^2 - 20x - 10} \\ 0 \end{array}$$

Thus, $(x + 1)^2(3x^2 - 13x - 10) = 0$ and if we factor $3x^2 - 13x - 10$, we have

$$(x + 1)^2(3x + 2)(x - 5) = 0$$

The solution set is $\left\{-1, -\frac{2}{3}, 5\right\}$.

▶ ***Quick check*** Given the equation $2x^4 + 5x^3 - 51x^2 + 80x - 28 = 0$ has a solution 2 of multiplicity 2, find the solution set of the equation.
Find a polynominal $P(x)$ of lowest degree whose zeros are 3, -2, and 1 of multiplicity 2.

Mastery points

Can you

- Divide $P(x)$ by $x - r$ using synthetic division?
- Use the remainder theorem to find the remainder R when polynomial $P(x)$ is divided by polynomial divisor $x - r$?
- Use the factor theorem to determine if polynomial $x - r$ is a factor of $P(x)$?
- Use the factor theorem to solve an equation $P(x) = 0$ where $P(x)$ is of degree greater than or equal to 3?
- Use the factor theorem to find an equation when given the real roots of the equation?
- Determine the multiplicity of a factor $x - r$?

Exercise 4–6

Perform the indicated divisions using synthetic division. Assume that no divisor is equal to zero. See example 4–6 A.

Example $(x^3 + 10x - 2) \div (x + 2)$

Solution Since there is no x^2 term, we insert $0x^2$ in the dividend to obtain $(x^3 + 0x^2 + 10x - 2) \div (x + 2)$.

$$\begin{array}{r|rrrrl} -2 & 1 & 0 & 10 & -2 & \leftarrow \text{Coefficients} \\ & \downarrow & -2 & 4 & -28 & \leftarrow \text{Add} \\ \hline & 1 & -2 & 14 & -30 & \end{array}$$

The answer is $x^2 - 2x + 14 - \dfrac{30}{x + 2}$.

1. $\dfrac{a^2 + 7a + 10}{a + 5}$
2. $\dfrac{x^2 + 7x + 6}{x + 1}$
3. $\dfrac{3a - 10 + a^2}{a - 2}$
4. $\dfrac{y^2 - 6 + y}{y - 2}$
5. $\dfrac{x^2 + 5x + 9}{x - 2}$
6. $\dfrac{y^2 - 6y - 4}{y - 3}$
7. $\dfrac{2a + 3a^2 - 1}{a + 1}$
8. $\dfrac{5 - 3x + 2x^2}{x + 2}$
9. $\dfrac{a^3 + 2a^2 - a - 4}{a - 1}$
10. $\dfrac{y^3 - 3y^2 + y + 2}{y + 1}$
11. $\dfrac{x^3 - 1}{x - 1}$
12. $\dfrac{a^3 + 1}{a + 1}$
13. $\dfrac{y^3 + 3y^2 - 4}{y - 2}$
14. $\dfrac{x^3 + 5x^2 - 1}{x - 1}$
15. $\dfrac{2a^3 - 9a^2 - 8a + 17}{a - 5}$
16. $\dfrac{3x^3 + 11x^2 + 12}{x + 4}$
17. $\dfrac{x^4 - 5x^3 + 5x^2 + 6x - 10}{x - 3}$
18. $\dfrac{2a^4 + 6a^3 + a^2 + 2a - 5}{a + 3}$
19. $\dfrac{3y^4 - 7y^3 - 10y^2 + 11y + 6}{y - 3}$
20. $\dfrac{a^5 + 2a^4 + 3a^3 + 6a^2 + a + 2}{a + 2}$

Given $P(x) = 3x^4 - 2x^3 + x^2 - 4x + 1$, in problems 21–26, find the following by (a) using the remainder theorem and (b) by substituting the indicated value of x. See example 4–6 B.

Example Using synthetic division and the remainder theorem, find the remainder when $P(x) = 3x^3 - 2x^2 + 4x - 5$ is divided by $(x + 4)$. Evaluate $P(-4)$ using substitution.

Solution a. Using synthetic division,

$$\begin{array}{r|rrrrl} -4 & 3 & -2 & 4 & -5 & \\ & & -12 & 56 & -240 & \\ \hline & 3 & -14 & 60 & -245 & \leftarrow \text{Remainder} \end{array}$$

b. $P(x) = 3x^3 - 2x^2 + 4x - 5$

$$\begin{aligned} P(-4) &= 3(-4)^3 - 2(-4)^2 + 4(-4) + 5 \qquad \text{Replace } x \text{ with } -4 \\ &= -192 - 32 - 16 - 5 \\ &= -245 \\ P(-4) &= -245 \end{aligned}$$

21. $P(1)$
22. $P(-1)$
23. $P(-3)$
24. $P(3)$
25. $P(0)$
26. $P(4)$

27. Given $P(x) = 5x^5 - 2x^3 + 1$, use the remainder theorem to find $P(2)$.

28. Given $P(x) = 8x^6 + 3x^2 - 2x - 5$, use the remainder theorem to find $P(-2)$.

29. Given $P(x) = 24x^4 - 16x^2 + 14x - 6$, use the remainder theorem to find $P\left(\frac{1}{2}\right)$.

Use the factor theorem to determine if the given binomial is a factor of the polynomial $P(x)$. See example 4–6 C–1.

Example Show that $x + 5$ is a factor of $P(x) = x^3 + 4x^2 - 7x - 10$.

Solution Using synthetic division,

$$\begin{array}{r|rrrr} -5 & 1 & 4 & -7 & -10 \\ & & -5 & 5 & 10 \\ \hline & 1 & -1 & -2 & 0 \end{array} \leftarrow x + 5 \text{ is a factor}$$

30. $P(x) = 3x^2 - 4x - 4$; $x - 2$

31. $P(x) = 2x^2 + 3x - 5$; $x + 3$

32. $P(x) = x^3 + x^2 - 7x - 10$; $x + 2$

33. $P(x) = x^3 - x^2 + 2x - 8$; $x - 2$

34. $P(x) = 4x^4 - 13x^3 - 13x^2 - 4x + 12$; $x + 2$

35. $P(x) = x^4 - 9x^3 + 18x^2 - 3$; $x + 1$

36. $P(x) = x^4 - 81$; $x - 3$

37. $P(x) = x^3 + 64$; $x + 4$

38. $P(x) = 3x^5 - 3x^4 + 5x^2 - 13x - 6$; $x - 3$

39. $P(x) = 3x^5 + 4x^2 - 7$; $x + 1$

Find the solution set of the following equations using the factor theorem and the given root. See example 4–6 C–2 and D–3.

Example Given the equation $2x^4 + 5x^3 - 51x^2 + 80x - 28 = 0$ has a solution 2 of multiplicity 2, find the solution set of the equation.

Solution Given a solution 2 of multiplicity 2, then $(x - 2)^2 = x^2 - 4x + 4$ is a factor of the equation. We divide $2x^4 + 5x^3 - 51x^2 + 80x - 28$ by $x^2 - 4x + 4$.

$$\begin{array}{r} 2x^2 + 13x - 7 \\ x^2 - 4x + 4\overline{)2x^4 + 5x^3 - 51x^2 + 80x - 28} \\ \underline{2x^4 - 8x^3 + 8x^2} \qquad\qquad\quad \\ 13x^3 - 59x^2 + 80x \qquad \\ \underline{13x^3 - 52x^2 + 52x} \qquad \\ -7x^2 + 28x - 28 \\ \underline{-7x^2 + 28x - 28} \\ 0 \end{array}$$

The quotient $2x^2 + 13x - 7 = (2x - 1)(x + 7)$, and if we set each factor equal to zero, we obtain $x = \frac{1}{2}$ and $x = -7$. The solution set is $\left\{-7, 2, \frac{1}{2}\right\}$.

40. $x^3 + 7x^2 - 2x - 12 = 0; -2$

41. $3x^3 + 19x^2 - 38x + 16 = 0; 1$

42. $x^3 - 5x^2 - 2x + 24 = 0; 3$

43. $2x^3 + x^2 - 61x + 30 = 0; 5$

44. $12x^3 + 29x^2 + 8x - 4 = 0; -2$

45. $x^4 - 2x^2 + 1 = 0$; 1 has multiplicity 2

46. $x^4 + 6x^3 - 3x^2 - 52x - 60 = 0$; -2 has multiplicity 2

Find a polynomial $P(x)$ of lowest degree having the given zeros. See example 4–6 C–3.

Example Find a polynomial $P(x)$ of lowest degree whose zeros are 3, -2, and 1 of multiplicity 2.

Solution Since $P(x)$ has a zero

1. 3, then $x - 3$ is a factor
2. -2, then $x + 2$ is a factor
3. 1 of multiplicity 2, then $(x - 1)^2 = x^2 - 2x + 1$ is a factor

Thus, $P(x) = (x - 3)(x + 2)(x^2 - 2x + 1) = x^4 - 3x^3 - 3x^2 + 11x - 6$.

47. 2, -1, 4

48. 5, -3, 3

49. 4, 0, -1

50. -2, 2, -3, 3

51. -8, 8, -1, 1

52. 4, -3, -7, 7

53. 5, -5, 6, 2

54. 3 of multiplicity 2; -3 of multiplicity 2

55. -1 of multiplicity 2; 4 of multiplicity 2

56. 0 of multiplicity 3; -2, 4

57. -5, -2, 0 of multiplicity 2

In problems 58–61, find the zeros of the given polynomial, indicating multiplicity where necessary. See example 4–6 D–1.

58. $P(x) = (x - 2)^2(x + 3)(x - 4)^2$

59. $P(x) = (x - 2)^2(x + 2)^2(x - 3)^3(x + 4)^2$

60. $P(x) = x^3(x - 3)^2(x + 5)$

61. $P(x) = x^2(x + 4)^4(x - 3)^2(x - 6)$

62. Show that $x - 2$ is a factor of $P(x) = x^{10} - 1024$.

63. Find the remainder when $4x^{98} - 8x^{47} + 9x^{28} - 3x^{17} + 1$ is divided by $x + 1$.

64. Find all values of k such that $P(x) = k^2x^3 - 7kx + 6$ is divisible by $x - 1$.

65. Use synthetic division to decide whether or not $3/2$ is a solution of the equation $2z^4 - 5z^3 + 11z^2 - 14z + 3 = 0$.

66. Use synthetic division to decide whether or not $1/3$ is a solution of the equation $6x^4 + x^3 - 4x^2 + 13x - 4 = 0$.

Review exercises

Find the solution set of the following equations. See section 2–1.

1. $5(x - 4) + 12 = 3x + 2x - 6$

2. $\frac{1}{2}x - 1 = \frac{1}{3}x + 3$

3. $\frac{7}{12}y - 4 = \frac{5}{6}y - 5$

4. One number is 27 more than another number. The lesser number is one-fourth of the greater number. Find the two numbers. See section 2–3.

5. Solve $V = \frac{1}{3}\pi h^2(3R - h)$ for R. See section 2–2.

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4–7 ■ Equations containing rational expressions

Rational equations

In chapter 2, we studied how to find the solution set of a linear equation. We now consider equations that contain at least one term that is a rational expression. We call these equations **rational equations.** The same properties of real numbers that we used in chapter 2 will apply to solve rational equations once the polynomial denominators are eliminated.

Recall that an equivalent equation is obtained when each term of an equation is multiplied by the same nonzero number. The following examples demonstrate the use of this property to solve rational equations.

■ Example 4–7 A

Find the solution set of the following rational equation.

1. $\dfrac{x}{3} + \dfrac{x}{4} - \dfrac{x}{6} = 35$

The LCM of 3, 4, and 6 is 12.

$$\begin{aligned} 12 \cdot \frac{x}{3} + 12 \cdot \frac{x}{4} - 12 \cdot \frac{x}{6} &= 12 \cdot 35 && \text{Multiply each term by the LCM 12} \\ 4x + 3x - 2x &= 420 && \text{Simplify each term} \\ 5x &= 420 && \text{Combine like terms} \\ x &= 84 && \text{Divide by 5} \end{aligned}$$

The solution set is $\{84\}$.

When the rational equation contains rational expressions with the variable in the denominator, multiplying all terms by the LCM of the denominators does *not always* produce an equation that is equivalent to the original equation.

2. $1 - \dfrac{2a}{a+1} = \dfrac{2}{a+1}$

$$\begin{aligned} (a+1) \cdot 1 - (a+1) \cdot \frac{2a}{a+1} &= (a+1) \cdot \frac{2}{a+1} && \text{Multiply each term by the LCM } (a+1) \\ a + 1 - 2a &= 2 && \text{Reduce in each term} \\ 1 - a &= 2 && \text{Combine like terms} \\ -a &= 1 && \text{Subtract 1 from each member} \\ a &= -1 && \text{Multiply each member by } -1 \end{aligned}$$

However, in the original equation, -1 *is not in the domain of the rational expressions involved.* Therefore -1 cannot be a solution and we conclude that the equation has no solution. Hence the solution set is $\emptyset$.

► ***Quick check*** Find the solution set of $3 - \dfrac{4x}{x-2} = \dfrac{-8}{x-2}$. ■

Extraneous solutions

The possible solution -1 in example 2 is called an **extraneous solution**—a solution of the equation obtained by multiplying each term by the LCM of the denominators but *not a solution of the original equation.*

Note It is important to observe the domain of the rational expressions in the rational equation to know where extraneous solutions may occur.

To further illustrate, consider the rational equation

$$\frac{3x}{x-3} = 2 + \frac{9}{x-3}$$

Note that the domain of the rational expressions is all real numbers *except 3.* That is, $x \neq 3$. If we proceed to solve the equation, we *do* obtain $x = 3$. Thus 3 is an extraneous solution, the equation has no solution, and the solution set is $\emptyset$.

The procedure for solving rational equations is stated here.

To solve rational equations

1. Find the LCM of the denominators.
2. Multiply each term by the LCM of the denominators.
3. Solve the resulting equation.
4. Check for extraneous solutions if the LCM contains a variable.

Equations and formulas containing rational expressions and more than one variable are common in scientific fields. It is often desirable to solve such equations for one variable in terms of the other variables in the equation. We use the same procedure that we used in solving preceding equations of this section.

Example 4–7 B

Solve each of the following equations for the indicated variable. Assume all denominators are nonzero.

1. $\frac{3x}{5} - \frac{4y}{3} = 6$ for x

$$15 \cdot \frac{3x}{5} - 15 \cdot \frac{4y}{3} = 15 \cdot 6$$ Multiply by the LCM of 5 and 3, namely 15

$$3 \cdot 3x - 5 \cdot 4y = 90$$ Simplify each term

$$9x - 20y = 90$$

$$9x = 90 + 20y$$ Add 20y to each member

$$x = \frac{90 + 20y}{9}$$ Divide each member by 9

2. $\frac{3}{x} - 7 = 9y + \frac{2y}{3x}$ for y

$$3x \cdot \frac{3}{x} - 3x \cdot 7 = 3x \cdot 9y + 3x \cdot \frac{2y}{3x}$$ Multiply each term by LCM of x and 3x, 3x

$$3 \cdot 3 - 21x = 27xy + 2y$$ Simplify each term

$$9 - 21x = 27xy + 2y$$

Since we cannot combine the terms containing y in the right member, we must *factor* the common factor, y, from each term.

$$9 - 21x = (27x + 2)y \quad \text{Factor } y \text{ in the right member}$$

$$\frac{9 - 21x}{27x + 2} = y \quad \text{Divide each member by the coefficient of } y, (27x + 2)$$

3. In physics, the rule governing the speeds of two gears, one the driver gear A and the other the driven gear B, is given by

$$\frac{T_A}{T_B} = \frac{R_B}{R_A}$$

where T = the number of teeth in the gear and R = the revolutions per minute of the gear. Solve for the revolutions per minute of the driven gear, R_B.

$$T_BR_A \cdot \frac{T_A}{T_B} = T_BR_A \cdot \frac{R_B}{R_A} \quad \text{Multiply each term by the LCM of } T_B \text{ and } R_A, T_BR_A$$

$$R_A \cdot T_A = T_B \cdot R_B \quad \text{Reduce in each term}$$

$$R_B = \frac{R_AT_A}{T_B} \quad \text{Divide each member by } T_B \text{ and interchange the two members}$$

▶ ***Quick check*** Given $\frac{6}{p-3} + \frac{2}{pq} = \frac{5}{4p}$, solve for p.

Mastery points

Can you

- Solve a rational equation in one variable?
- Solve a rational equation in two or more variables for one of the variables?

Exercise 4–7

Find the solution set of each of the following equations. Where extraneous solutions exist, so state. See example 4–7 A.

Example Find the solution set of $3 - \frac{4x}{x-2} = \frac{-8}{x-2}$.

Solution The LCM is $(x - 2)$.

$$(x-2)3 - (x-2)\frac{4x}{x-2} = (x-2)\frac{-8}{x-2} \quad \text{Multiply each term by LCM } x - 2$$

$$3x - 6 - 4x = -8 \quad \text{Reduce and multiply}$$

$$-x - 6 = -8 \quad \text{Combine like terms}$$

$$-x = -2 \quad \text{Add 6 to each member}$$

$$x = 2 \quad \text{Multiply by } -1$$

Since the domain of the rational expressions is all real numbers except 2, the solution set is $\emptyset$ and 2 is an extraneous solution.

1. $\frac{x+7}{4} = \frac{2x}{12}$

2. $\frac{4b-3}{10} = \frac{2b-1}{6}$

3. $\frac{3x}{5} - \frac{4x}{3} = 1$

4. $\frac{m}{3} + 4 = \frac{7m}{4}$

$\boxed{5.}$ $\frac{4}{a} - \frac{6}{3a} = \frac{3}{5}$

6. $4 - \frac{5}{9b} = \frac{7}{6b}$

7. $\frac{b-3}{10} + \frac{2b+1}{15} = 2$

8. $\frac{3y-4}{16} - \frac{2-3y}{12} = 1$

9. $\frac{x-3}{3x} = \frac{2x+3}{9x}$

10. $\frac{5-b}{8b} = \frac{3b+7}{6b}$

11. $\frac{5}{x-2} = \frac{4}{2x+1}$

12. $\frac{-3}{x+7} = \frac{2}{3x-1}$

13. $\frac{5}{4a+2} = \frac{7}{2a+1}$

14. $\frac{11}{3y-2} = \frac{8}{12y-8}$

15. $\frac{6}{6x+3} = \frac{3}{2x+1} + 5$

$\boxed{16.}$ $1 + \frac{5}{3m-9} = \frac{10}{m-3}$

17. $\frac{x-1}{x^2-4} = \frac{6}{x-2}$

18. $\frac{5}{y+3} = \frac{2y+1}{y^2-9}$

19. $\frac{x}{x-2} + \frac{2}{3} = \frac{2}{x-2}$

20. $\frac{3}{2} - \frac{1}{x-4} = \frac{-2}{2x-8}$

$\boxed{21.}$ $4 - \frac{2x}{5-x} = \frac{6}{x-5}$

22. $\frac{5y}{3-y} + \frac{8}{y-3} = 4$

23. $\frac{8b}{b^2-16} = \frac{3}{b+4} + \frac{5}{4-b}$

24. $\frac{10}{5-a} = \frac{7}{a+5} - \frac{6}{a^2-25}$

25. $\frac{5}{a-3} - \frac{1}{a+2} = \frac{6a}{a^2-a-6}$

26. $\frac{7}{x+6} = \frac{9}{x-9} - \frac{2x-1}{x^2-3x-54}$

27. $\frac{4}{2x-6} - \frac{12}{4x+12} = \frac{12}{x^2-9}$

28. $\frac{11x-3}{10x^2-3x-4} - \frac{4}{5x-4} = \frac{9}{2x+1}$

$\boxed{29.}$ $\frac{6}{q^2+q-6} = \frac{5}{q^2+3q-10}$

30. $\frac{9}{y^2-6y+8} = \frac{14}{y^2-16}$

31. $\frac{13}{n^2+2n-15} - \frac{1}{n^2+10n+25} = 0$

32. $\frac{2}{2n^2-7n-4} + \frac{6}{6n^2-5n-4} = 0$

33. $\frac{4}{x^2-9} = \frac{7}{x^2-7x+12} - \frac{5}{x^2-x-12}$

34. $\frac{-2}{a^2+3a-4} - \frac{6}{a^2-1} = \frac{1}{a^2+5a+4}$

35. $\frac{6}{2n^2+n-3} - \frac{5}{4n^2-9} = \frac{6}{2n^2-5n+3}$

36. $\frac{6}{8x^2-14x-15} + \frac{1}{8x^2-26x+15} = \frac{9}{16x^2-9}$

Solve the given equations and formulas for the indicated variable. Assume all denominators are nonzero. See example 4–7 B.

Example Given $\frac{6}{p-3} + \frac{2}{pq} = \frac{5}{4p}$, solve for p.

Solution The LCM of $4p$, pq, and $p - 3$ is $4pq(p - 3)$.

$$4pq(p-3)\frac{6}{p-3} + 4pq(p-3)\frac{2}{pq} = 4pq(p-3)\frac{5}{4p}$$ Multiply each term by LCM $4pq(p-3)$

$$4pq \cdot 6 + 4(p-3)2 = q(p-3)5$$ Reduce in each term

$$24pq + 8(p-3) = 5q(p-3)$$ Commute and multiply

$$24pq + 8p - 24 = 5pq - 15q$$ Distributive property

$$19pq + 8p = 24 - 15q$$ Add 24 to and subtract $5pq$ from each member

$$(19q + 8)p = 24 - 15q$$ Factor p in right member

$$p = \frac{24-15q}{19q+8}$$ Divide each member by $19q + 8$

$$p = \frac{3(8-5q)}{19q+8}$$ Factor numerator (not reducing)

37. $\frac{4}{a} - \frac{3}{b} = 7$ for b

38. $\frac{6}{x} - 4 = \frac{7}{y}$ for y

39. $\frac{3}{p} + 4 = \frac{6q}{2p} - 3a$ for p

40. $\frac{a+6}{3} - \frac{b-2}{4} = \frac{c}{12}$ for a

41. $\frac{y-3}{x+2} = \frac{5}{3}$ for y

42. $\frac{-7}{3} = \frac{y+1}{x-5}$ for x

43. $S = \frac{a}{1-r}$ for r (progression formula)

44. $S = \frac{n(a+b)}{2}$ for a (progression formula)

45. $A = \frac{1}{2}h(b_1 + b_2)$ for b_1 (area of a trapezoid)

46. $C = \frac{5}{9}(F - 32)$ for F (temperature conversion)

See example 4–7 B–3.

47. The coefficient of linear expansion, k, of a solid when heated is given by

$$k = \frac{L_t - L_0}{L_0 t}$$

where L_t is the length at $t\,°C$, L_0 is the length at $0°C$, and t is any given temperature in Celsius. Solve for t. Solve for L_0.

48. A formula for unknown resistance R_x in a battery is given by

$$R_x = R_m\left(\frac{E_1}{E_2} - 1\right)$$

Solve for E_1.

49. The interest rate, r, on a given amount of money, P, over a given period of time, t, which pays interest, I, is given by

$$r = \frac{I}{Pt}$$

Solve the equation for t.

50. The kinetic energy of a body, KE, is computed by

$$KE = \frac{Wv^2}{2g}$$

where W is the weight in pounds, v is the velocity expressed in feet per second, and g is the acceleration due to gravity. Solve for g.

51. The total resistance R of a parallel circuit is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

where R_1, R_2, and R_3 are the resistances of the respective circuits. Solve for R_3.

52. The capacitance of capacitors C_1, C_2, and C_3 in a series circuit is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Solve for C.

53. Given a large piston and a small piston on which forces F and f, respectively, are applied, then

$$\frac{F}{f} = \frac{A}{a}$$

where A is the area of the large piston and a is the area of the small piston. Solve for A.

54. As gas expands when heated, the relationship between pressure P, volume V, and absolute temperature T is given by

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

called Charles' Law. Solve for V_2.

55. The formula

$$\frac{R_2}{R_1} = \frac{M + T_2}{M + T_1}$$

gives a relationship for the increase in the resistance of a circuit caused by a rise in temperature. Solve for T_2. Solve for R_1.

56. The lens maker's formula for the focal length, f, of a lens is given by

$$\frac{1}{f} = (\mu_m - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

where μ_m is the index of refraction and R_1 and R_2 are radii of curvature. Solve for f. Solve for μ_m.

57. If Ann can paint the house in 10 hours, write a rational expression for what part of the house she can paint in (a) 1 hour, (b) 5 hours, (c) t hours.

58. If John can bake a batch of cookies in 3 hours, write a rational expression for what part of the batch he can bake in (a) 1 hour, (b) t hours, (c) 20 minutes.

59. An automobile travels 200 miles. Write a rational expression for the speed (rate of travel) the automobile is traveling if the distance is covered in (a) 5 hours, (b) 4 hours, (c) t hours. (*Hint:* distance = rate · time)

60. Write a rational expression for the time t a boat travels 20 miles downstream if the boat is traveling downstream at (a) 4 mph, (b) 12 mph, (c) r mph.

$\boxed{61.}$ If one number is three times as large as a second number, and if n represents the second number, (a) write a rational expression for the reciprocal of the two numbers, (b) write a rational expression for five times the reciprocal of the second number.

62. If a number n is added to the numerator and the denominator of the rational number $\frac{3}{5}$, write a rational expression for the new number.

63. If the same number n is added to the numerator and taken away from the denominator of $\frac{5}{6}$, write a rational expression for the new number.

Review exercises

Completely factor the following expressions. See sections 3–5, 3–6, and 3–7.

1. $4p^2 - 25q^2$
2. $x^2 - 24x + 144$
3. $2y^2 - 15y - 8$
4. Dene invests \$20,000, part at 6% and the rest at 8% interest. If the total income from the investments in one year is \$1,360, how much did she invest at each rate? See section 2–3.
5. Five times a number increased by 13 gives 53. Find the number. See section 2–3.
6. Given polynomial $P(x) = 2x^2 + x - 3$, find $P(-3)$. See section 1–5.

4–8 ■ Problem solving with rational equations

Now that we can solve rational equations, let us use that skill to solve some types of word problems that result in rational equations when we use problem-solving techniques.

■ Example 4–8 A

Work problem

1. Jan can produce a part in 3 hours and Joe can produce the same part in 4 hours. How long would it take them to produce the part working together?

   Since Jan can produce the part in 3 hours and Joe can produce the part in 4 hours, then Jan can produce $\frac{1}{3}$ of the part in 1 hour and Joe can produce $\frac{1}{4}$ of the part in 1 hour.

   Let x = the time in hours necessary to produce the part working together.

Then they can produce $\frac{1}{x}$ of the part in 1 hour.

$$\underset{\text{Jan in 1 hour}}{\frac{1}{3}} + \underset{\text{Joe in 1 hour}}{\frac{1}{4}} = \underset{\text{together in 1 hour}}{\frac{1}{x}}$$

The LCM of the denominators is $12x$.

$$12x \cdot \frac{1}{3} + 12x \cdot \frac{1}{4} = 12x \cdot \frac{1}{x} \qquad \text{Multiply each term by the LCM } 12x$$
$$4x + 3x = 12 \qquad \text{Reduce in each term}$$
$$7x = 12 \qquad \text{Add in left member}$$
$$x = \frac{12}{7} \text{ or } 1\frac{5}{7} \text{ hours} \qquad \text{Divide by 7}$$

Working together, Jan and Joe can produce the part in $1\frac{5}{7}$ hours.

Uniform motion problems

2. An automobile can travel 200 miles in the same time that a truck can travel 150 miles. If the automobile travels at an average rate of 15 miles per hour faster than the truck, find the average rate of each. We use the relationship between distance traveled, rate of travel, and time traveled—distance (d) = rate (r) · time (t).

The following table will be helpful in finding the equation necessary to solve this type of problem.

	d =	r ×	t
Automobile			
Truck			

Let r = the rate of the truck. Then $r + 15$ = the rate of the automobile.

	d =	r ×	t
Automobile	200	$r + 15$	$\frac{200}{r + 15}$
Truck	150	r	$\frac{150}{r}$

Times are equal

Note To find the time, we used the relationship

$$\text{time} = \frac{\text{distance}}{\text{rate}}.$$

Since both vehicles traveled the same length of time, we obtain the equation

$$\frac{200}{r+15} = \frac{150}{r}$$

$$\begin{aligned}
r(r+15)\frac{200}{r+15} &= r(r+15)\frac{150}{r} && \text{Multiply each term by the LCM of } r \text{ and } (r+15),\ r(r+15) \\
200r &= (r+15)150 \\
200r &= 150r + 2{,}250 \\
50r &= 2{,}250 \\
r &= 45 \\
r + 15 &= 60
\end{aligned}$$

Thus the average rate of the truck is 45 miles per hour and of the automobile is 60 miles per hour.

3. John's boat travels at 20 miles per hour in still water. If the speed of the current is 5 miles per hour, how far can John travel downstream if it takes him a total time of 3 hours to go downstream and back upstream?

Let x = the distance traveled downstream (and back upstream since they are the same distance). Using $t = \dfrac{d}{r}$, we summarize the information in the table.

	d	r	t
Downstream	x	$20 + 5 = 25$	$\dfrac{x}{25}$
Upstream	x	$20 - 5 = 15$	$\dfrac{x}{15}$

Total time is 3 hours

Note When traveling downstream, we *add* the speed in still water to the speed of the current. When traveling upstream, we *subtract* the speed of the current from the speed in still water.

Since total time downstream and back upstream is 3 hours, the equation is

$$\underbrace{\frac{x}{25}}_{\text{Downstream}} + \underbrace{\frac{x}{15}}_{\text{Upstream}} = 3 \leftarrow \text{Total time}$$

$$\begin{aligned}
\frac{x}{25}\cdot 75 + \frac{x}{15}\cdot 75 &= 3\cdot 75 && \text{Multiply each term by the LCM of 25 and 15, 75} \\
3x + 5x &= 225 && \text{Multiply as indicated} \\
8x &= 225 && \text{Combine like terms} \\
x &= \frac{225}{8} && \text{Divide each member by 8} \\
&= 28\frac{1}{8}
\end{aligned}$$

John can travel $28\frac{1}{8}$ miles downstream.

Number and reciprocal problem

4. One number is twice another number. The sum of their reciprocals is $\frac{9}{2}$. Find the numbers.

Let x = one number, then $2x$ = the other number. Their reciprocals are then $\frac{1}{x}$ and $\frac{1}{2x}$. The sum of the reciprocals is $\frac{9}{2}$, so we have the equation

$$\frac{1}{x} + \frac{1}{2x} = \frac{9}{2}$$

$$2x \cdot \frac{1}{x} + 2x \cdot \frac{1}{2x} = 2x \cdot \frac{9}{2} \qquad \text{Multiply each term by the LCM of } x,\ 2, \text{ and } 2x,\ 2x$$

$$2 + 1 = 9x$$
$$3 = 9x$$
$$x = \frac{3}{9} = \frac{1}{3}$$
$$2x = \frac{2}{3}$$

The two numbers are $\frac{1}{3}$ and $\frac{2}{3}$.

Check: Since the sum of the reciprocals $3 + \frac{3}{2} = \frac{6}{2} + \frac{3}{2} = \frac{9}{2}$, the answer is correct.

▶ ***Quick check*** Pam can clean her house in 2 hours. Her daughters, Linnea and Jean, can do the job in 2 hours and 3 hours, respectively. At these rates, how long would it take the three of them to clean the house working together?

Mastery points

Can you

- Set up and solve work problems?
- Set up and solve uniform motion problems?
- Set up and solve number and reciprocal problems?

Exercise 4–8

Solve the following work problems. See example 4–8 A–1.

Work problems

Example Pam can clean her house in 2 hours. Her daughters, Linnea and Jean, can do the job in 2 hours and 3 hours, respectively. At these rates, how long would it take the three of them to clean the house working together?

Solution Let x = time in hours to clean the house when the three of them are working together.

Then 1. Pam can clean $\frac{1}{2}$ of the house in 1 hour.

2. Linnea can clean $\frac{1}{2}$ of the house in 1 hour.

3. Jean can clean $\frac{1}{3}$ of the house in 1 hour.

4. Working together, they can clean $\frac{1}{x}$ of the house in 1 hour.

$$\underbrace{\frac{1}{2}}_{\text{part done by Pam}} + \underbrace{\frac{1}{2}}_{\text{part done by Linnea}} + \underbrace{\frac{1}{3}}_{\text{part done by Jean}} = \underbrace{\frac{1}{x}}_{\text{part done by all three working together}}$$

The equation is $\frac{1}{2} + \frac{1}{2} + \frac{1}{3} = \frac{1}{x}$.

$$6x \cdot \frac{1}{2} + 6x \cdot \frac{1}{2} + 6x \cdot \frac{1}{3} = 6x \cdot \frac{1}{x} \qquad \text{Multiply each term by the LCM of 2, 3, and } x, 6x$$

$$3x + 3x + 2x = 6$$

$$8x = 6 \qquad \text{Combine like terms}$$

$$x = \frac{6}{8} \qquad \text{Divide each member by 8}$$

$$x = \frac{3}{4} \qquad \text{Simplify}$$

It would take Pam and her daughters $\frac{3}{4}$ of an hour to clean the house working together.

1. Pete Hansen milks his herd of cattle in 2 hours, whereas his son John milks them in 3 hours. How many hours and minutes would it take them to do the same job working together?

2. Jake has two hay balers, A and B. If Jake can completely bale a given field in 3 hours using baler A and in $3\frac{3}{4}$ hours using baler B, how long would it take him to bale the field working both balers together?

3. During a "clean-up, paint-up, fix-up week" in Sault Sainte Marie, Jim, Toni, and Ken clean up a given vacant lot. If it takes each person $1\frac{1}{2}$ hours, 2 hours, and 3 hours, respectively, to do the job alone, how long would it take them working together to clean up the lot?

4. Tanya, Bill, and Neysa work in a bakery and can mix 12 loaves of bread individually in 36 minutes, 40 minutes, and 30 minutes, respectively. If they worked together, in how many minutes could they mix the 12 loaves?

5. If two water pipes can fill a swimming pool in 18 hours when both pipes are open and one of the pipes can fill the pool alone in 30 hours, how long would it take the other pipe to fill the pool alone?

6. In a machine shop, machine A can produce certain parts in 1 hour and 20 minutes. If machines A and B working together can produce the parts in 50 minutes, how long would it take machine B to do the job alone?

7. Two inlet pipes can fill a water basin in 10 hours and 12 hours, respectively, when open individually. If an outlet pipe can empty the basin alone in 9 hours, how long would it take to fill the basin if all three pipes are open simultaneously?

8. An open sink drain can empty a sink full of water in 2 minutes. If the cold water and hot water faucets can, when open fully, fill the sink in $3\frac{1}{2}$ and 3 minutes, respectively, how long would it take to fill the sink if all three are open simultaneously?

9. It takes pump A twice as long to unload an oil tanker as it takes pump B to do the job. If the two pumps working together can unload the tanker in 12 hours, how long will it take each to do the job?

$\boxed{10.}$ If one microprocessor can process a set of inputs in three-fifths of the time that a second microprocessor can do the job, and together the machines can do the job in 2 milliseconds, how long will it take each microprocessor to process the set of inputs individually?

11. It takes tug A one-half as long to push a series of barges up a river as it takes tug B to do the same job. If it takes tug C two times as long to do the job as it does tug B and all three tugs working together can do the job in 4 hours, how long would it take tug A to do the job alone?

12. A portable gasoline-powered generator, a solar cell, and a wind generator fully charge a dead storage battery in 9 hours when charging simultaneously. If the wind generator takes twice as long to fully charge the battery as the solar cell takes and the gasoline-powered generator takes three-fourths as long as the solar cell, how long would it take the solar cell to fully charge the battery alone?

Solve the given uniform motion problems. See example 4–8 A–2 and 3.

Uniform motion problems

13. An excursion boat moves at 16 miles per hour in still water. If the boat travels 20 miles downstream in the same time it takes to travel 14 miles upstream, what is the speed of the current? (*Hint:* Let x = the speed of the current. Then $16 + x$ = the speed of the boat downstream and $16 - x$ = the speed of the boat upstream.)

$\boxed{14.}$ An airplane can cruise at 300 miles per hour in still air. If the airplane takes the same time to fly 950 miles with the wind as it does to fly 650 miles against the wind, what is the speed of the wind?

15. A boat travels 40 kilometers upstream in the same time that it takes the same boat to travel 60 kilometers downstream. If the stream is flowing at 6 kilometers per hour, what is the speed of the boat in still water?

$\boxed{16.}$ The speed of a wind is 15 miles per hour. Find the speed of an airplane in still air if it flies 200 miles against the wind in the same time that it flies 300 miles with the wind.

17. On a trip from Detroit to Los Angeles, a TWA plane takes one-third of an hour longer to make the trip than an American Airlines plane does. If the TWA plane flies at 300 miles per hour and the American Airlines plane flies at 320 miles per hour, how far is it from Detroit to Los Angeles?

18. It takes Bonnie 2 minutes longer to jog a certain distance than it does Maryann. What distance did they run if Bonnie can jog at 5 miles per hour and Maryann can jog at 7 miles per hour?

19. Mary can row her boat 3 miles per hour in still water. If the current of the river is 2 miles per hour, how far downstream can she row if it takes her 2 hours to go down and back?

20. Peter can average 10 miles per hour riding his bike to deliver his papers. By car he can average 30 miles per hour. If it takes him $\frac{1}{2}$ hour less time by car, how long is Peter's paper route?

Solve the following number and reciprocal problems. See example 4–8 A–4.

Number and reciprocal problems

21. One number is three times another number. The sum of their reciprocals is 2. Find the numbers.

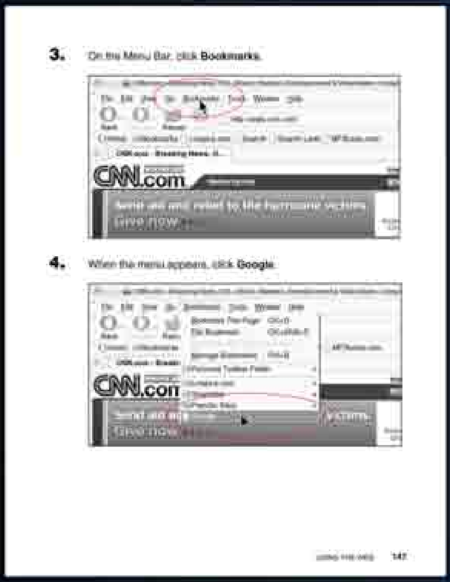
22. One number is twice another number. The sum of their reciprocals is $\frac{15}{8}$. Find the numbers.

23. If the same number is added to the numerator and the denominator of the fraction $\frac{3}{7}$, the result is $\frac{4}{5}$. Find the number.

24. If the same number is added to the numerator and the denominator of the fraction $\frac{1}{4}$, the result is $\frac{2}{3}$. Find the number.

25. What number must be added to the denominator of $\frac{6}{7}$ to obtain $\frac{3}{5}$?

26. What number must be added to the numerator and subtracted from the denominator of $\frac{5}{8}$ to obtain its reciprocal?


3. On the Menu Bar, click **Bookmarks**.

4. When the menu appears, click **Google**.

Page from a typical Visibook:
12 words, two large pictures.

The following word problems are a mixture of the previous types.

$\boxed{27.}$ A boat can go 6 miles upstream in the same time as it can go 10 miles downstream. If the boat goes 12 miles per hour in still water, what is the speed of the current?

28. What number must be added to the numerator of the fraction $\frac{11}{15}$ to obtain the fraction $\frac{7}{5}$?

29. Jane can type an English term paper in 3 hours. If Pamela helps her, together they can type the paper in 2 hours. How long would it take Pamela to type the paper alone?

$\boxed{30.}$ A water tank can be filled by its inlet pipe in 45 minutes. If its outlet pipe can empty the tank in 30 minutes, how long would it take to empty a full tank if both pipes are open?

31. Fran traveled 120 miles to visit her aunt. If she drove twice as fast returning home and the return trip took 2 hours less time, what was her speed going to her aunt's?

32. If one number is three times a second number and the sum of their reciprocals is $\frac{2}{9}$, find the numbers.

33. Erin can jog 2 miles per hour faster than her cousin Sarah. Erin can jog 9 miles in the same time Sarah can jog 6 miles. How fast does each girl jog?

Review exercises

Perform the indicated operations. See section 3–2.

1. $4x(2x^2 - 3x + 1)$ **2.** $(3x - 1)(x - 5)$ **3.** $(5z - 4)^2$ **4.** $(2y + 3)(2y - 3)$

Simplify the following expressions. Express the answer with positive exponents. See section 3–3.

5. $y^{-3} \cdot y^2 \cdot y^0$

6. $\dfrac{x^3y^2}{x^{-1}y^{-1}}$

7. What number(s) when squared yield (yields) 49? See section 3–1.

8. What number cubed yields -125? See section 3–1.

Chapter 4 lead-in problem

Doug Nance opens the valve to fill his new swimming pool. Without his knowlege, the drain to empty the pool has been left open. If the pool would normally fill in 10 hours and the drain can empty the full pool in 15 hours, how long does it take to fill the pool?

Solution

Let t represent the time to fill the pool. In 1 hour,

a. the valve would fill $\frac{1}{10}$ of the pool.

b. the drain would empty $\frac{1}{15}$ of the pool.

c. with both open, $\frac{1}{t}$ of the pool would be filled.

We *subtract* the drain time from the fill time to get the equation

$$\frac{1}{10} - \frac{1}{15} = \frac{1}{t}$$

$$30t\left(\frac{1}{10} - \frac{1}{15}\right) = 30t \cdot \frac{1}{t} \quad \text{Multiply each member by the LCD } 30t$$

$$30t\left(\frac{1}{10}\right) - 30t\left(\frac{1}{15}\right) = 30 \quad \text{Distribute in left member and multiply}$$

$$3t - 2t = 30 \quad \text{Simplify in left member}$$

$$t = 30 \quad \text{Combine like terms}$$

It takes 30 hours to fill the pool.

Chapter 4 summary

1. A **rational expression** is any algebraic expression that can be written as a quotient of two polynomials where the denominator does not equal zero.
2. The **domain** of a rational expression is the set of all replacement values of the variable for which the expression is defined.
3. The **fundamental principle of rational expressions** states that we obtain an equivalent expression when we multiply or divide the numerator and the denominator of a rational expression by the same nonzero polynomial.
4. A rational expression is **reduced to its lowest terms** when the numerator and the denominator have no common polynomial factors other than 1 or -1.
5. To **multiply** two or more rational expressions, multiply the numerators to determine the numerator of the product and multiply the denominators to determine the denominator of the product.
6. Given polynomials P, Q, R, and S,
   $$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} = \frac{P \cdot S}{Q \cdot R}$$
   where $Q \neq 0$, $R \neq 0$, and $S \neq 0$.
7. Given polynomials P, Q, and R, $Q \neq 0$,
   $$\frac{P}{Q} + \frac{R}{Q} = \frac{P+R}{Q}$$
   and
   $$\frac{P}{Q} - \frac{R}{Q} = \frac{P-R}{Q}$$
8. Given polynomials P, Q, R, and S, $Q \neq 0$ and $S \neq 0$,
   $$\frac{P}{Q} + \frac{R}{S} = \frac{P \cdot S + Q \cdot R}{Q \cdot S}$$
   and
   $$\frac{P}{Q} - \frac{R}{S} = \frac{P \cdot S - Q \cdot R}{Q \cdot S}$$
9. To find the least common multiple (LCM) of a set of polynomials,
   a. completely factor the polynomials,
   b. the LCM consists of the product of all distinct factors of the polynomials, each raised to the greatest power to which it appears in any of the factorizations.
10. Rational expressions can be added or subtracted by building each expression to an equivalent rational expression such that they have a common denominator—the least common denominator (LCD).
11. A **complex rational expression** is a rational expression whose numerator or denominator, or both, contain rational expressions.
12. In the complex rational expression
    $$\frac{\frac{4}{5}}{\frac{7}{8}}$$
    we call $\frac{4}{5}$ the **primary numerator**, $\frac{7}{8}$ the **primary denominator**, and 5 and 8 the **secondary denominators.**
13. To simplify a complex rational expression, multiply the primary numerator and the primary denominator by the least common multiple (LCM) of the secondary denominators.
14. To *divide* a polynomial by a monomial, simply divide each term of the polynomial by the monomial and add or subtract the resulting quotients.
15. When the divisor is a binomial of the form $x - k$, we can use synthetic division to divide a polynomial by a polynomial.
16. The **remainder theorem** and the **factor theorem** uses synthetic division to find (a) the remainder when a polynomial is divided by $x - k$ and (b) the solution set of a polynomial equation.
17. A **rational equation** is an equation that contains at least one expression of the form $\frac{P}{Q}$, $Q \neq 0$, where P and Q are polynomials.
18. To obtain a simple equation, an equation containing no rational expression, multiply both members of the rational equation by the LCM of the denominators.

Chapter 4 error analysis

1. Placing restrictions on the variable in a rational expression
   Example: What restrictions must be placed on the variable in $\dfrac{3y+2}{4y^2+3y}$?
   Since $4y^2 + 3y = 0$ when $y = 0$, then $y \neq 0$.
   Correct answer: $y \neq 0$ or $y \neq -\dfrac{3}{4}$
   What error was made? (*see page 155*)
2. Finding the domain of a rational expression
   Example: $\dfrac{2x-1}{2x^2+5x-3} = \dfrac{2x-1}{(2x-1)(x+3)} = \dfrac{1}{x+3}$
   Domain $= \{x \mid x \in R, x \neq -3\}$
   Correct answer: Domain $= \left\{x \mid x \in R, x \neq -3, \dfrac{1}{2}\right\}$
   What error was made? (*see page 155*)
3. Subtracting rational expressions
   Example: $\dfrac{4y+1}{y^2+3} - \dfrac{2y-4}{y^2+3}$
   $= \dfrac{4y+1-2y-4}{y^2+3} = \dfrac{2y-3}{y^2+3}$
   Correct answer: $\dfrac{2y+5}{y^2+3}$
   What error was made? (*see page 167*)
4. Adding rational expressions
   Example: $\dfrac{2x}{x-2} + \dfrac{x-7}{2-x} = \dfrac{2x+x-7}{x-2} = \dfrac{3x-7}{x-2}$
   Correct answer: $\dfrac{x+7}{x-2}$
   What error was made? (*see page 171*)
5. Simplifying complex rational expression to lowest terms
   Example: Simplify $\dfrac{1+\dfrac{1}{x}}{x-\dfrac{1}{x}}$.
   $$\frac{\left(1+\dfrac{1}{x}\right)\cdot x}{\left(x-\dfrac{1}{x}\right)\cdot x} = \frac{x+1}{x^2-1}$$
   Correct answer: $\dfrac{1}{x-1}$
   What error was made? (*see page 176*)
6. Reducing a rational expression to lowest terms
   Example: $\dfrac{2y^3+y}{y} = \dfrac{2y^3+\cancel{y}}{\cancel{y}} = \dfrac{2y^3+1}{1} = 2y^3+1$
   Correct answer: $2y^2 + 1$
   What error was made? (*see page 157*)
7. Dividing a polynomial by a polynomial
   Example: $(3x^3 + 2x^2 - x + 1) \div (x - 1)$
   $$\begin{array}{r} x^2 + x - 2 \\ x-1\overline{)\,x^3+2x^2-x+1} \\ \underline{x^3 - x^2} \\ x^2 - x \\ \underline{x^2 - x} \\ -2x+1 \\ \underline{-2x+2} \\ 3 \end{array} = x^2 + x - 2 + \frac{3}{x-2}$$
   Correct answer: $x^2 + 3x + 2 + \dfrac{3}{x-1}$
   What error was made? (*see page 185*)
8. Solving rational equations
   Example: Find the solution set of $\dfrac{3x}{x-4} = \dfrac{12}{x-4} + 1$.
   $(x-4)\cdot\dfrac{3x}{x-4} = (x-4)\cdot\dfrac{12}{x-4} + (x-4)\cdot 1$
   $3x = 12 + x - 4$
   $2x = 8$
   $x = 4 \quad \{4\}$
   Correct answer: The solution set is $\emptyset$.
   What error was made? (*see page 199*)
9. Using synthetic division
   Example: Divide $3x^3 - 2x^2 + 4x - 1$ by $x + 3$.
   $$\begin{array}{r|rrrr} 3 & 3 & -2 & 4 & -1 \\ & \downarrow & 9 & 21 & 75 \\ \hline & 3 & 7 & 25 & 74 \end{array} = 3x^2 + 7x + 25 + \frac{74}{x+3}$$
   Correct answer: $3x^2 - 11x + 37 - \dfrac{112}{x+3}$
   What error was made? (*see page 190*)
10. Finding the remainder using synthetic division
    Example: Given $P(x) = 4x^2 - 2x + 1$, find $P(3)$.
    $$\begin{array}{r|rrr} 3 & 4 & -2 & 1 \\ & \downarrow & 12 & -42 \\ \hline & 4 & -14 & 43 \end{array} \qquad P(3) = 43$$
    Correct answer: $P(3) = 31$
    What error was made? (*see page 191*)

Chapter 4 critical thinking

If n is an integer, for what values of n will $4n^2 + 4n - 3$ represent an odd integer?

Chapter 4 review

[4–1]

State the domain of the given rational expression in set-builder notation.

1. $\dfrac{5}{x+7}$
2. $\dfrac{2x+1}{3x-4}$
3. $\dfrac{4-2x}{x^2-10x+25}$
4. $\dfrac{4a^2+1}{9a^2-16}$
5. $\dfrac{3z-1}{6z^2+13z-5}$
6. $\dfrac{4y-11}{9y^2-12y+4}$

Reduce each rational expression to lowest terms. Assume all denominators are nonzero.

7. $\dfrac{a^3b^4c}{a^2bc^3}$
8. $\dfrac{-10m^3n^3p}{35m^4np^5}$
9. $\dfrac{5x-10}{6x-12}$
10. $\dfrac{24a}{8a^2-16a}$
11. $\dfrac{5x-10y}{4y^2-x^2}$
12. $\dfrac{y^3-64}{y^2-16}$
13. $\dfrac{a^2-24a+144}{a^2-11a-12}$
14. $\dfrac{4x^2-5x-6}{5x^2-11x+2}$
15. $\dfrac{10y^2+9y-9}{12-2y-30y^2}$

[4–2]

Perform the indicated multiplication or division. Place necessary restrictions on the variables.

16. $\dfrac{14x}{3y}\cdot\dfrac{9y^2}{7x^2}$
17. $\dfrac{16a^2}{7y}\div\dfrac{4a}{21y^2}$
18. $\dfrac{p^2-16}{4p-3}\cdot\dfrac{16p^2-9}{3p+12}$
19. $\dfrac{z^2+1}{2z-6}\div\dfrac{z^4-1}{z^2-6z+9}$
20. $\dfrac{m^3-8}{m^2-3m}\cdot\dfrac{m^2+3m-18}{m^2+3m-10}$
21. $\dfrac{5a^2+17a+6}{10a^2-a-2}\cdot\dfrac{2a^2+13a-7}{a^2+10a+21}$
22. $\dfrac{x^2-49}{8x^3+1}\div\dfrac{x^2-5x-14}{4x^2-1}$
23. $\dfrac{4x^3-5x^2}{4x+5}\div(16x^2-25)$
24. $\dfrac{7y^2+10y-8}{y^2+4y+4}\cdot\dfrac{y^2+6y+9}{7y^2+17y-12}$
25. $\dfrac{mq-mp-nq+np}{2mq-2mp-nq+np}\div\dfrac{qm+qn+pm+pn}{2mp-np+2qm-nq}$

[4–3]

Find the least common multiple (LCM) of the set of polynomials.

26. $15xy^2, 20x^3y, 9x^2y^3$
27. $2x+4, 6x^2, x^2-16$
28. $a^2+3a-10, a^2-2a, 3a+15$
29. $p^2-25, p^2+10p+25, p^2-5p$

Perform the indicated additions and/or subtractions. Assume all denominators are nonzero.

30. $\dfrac{3x}{4y}+\dfrac{8x}{3y}$
31. $\dfrac{n-7}{n+4}-\dfrac{3n+8}{n-1}$
32. $\dfrac{10}{p^2+7p-18}+\dfrac{9}{p^3-4p}$
33. $(2b+7)-\dfrac{b+9}{3b-2}$
34. $\dfrac{3y-4}{y^2-49}+\dfrac{2y+1}{y-7}$
35. $\dfrac{2x}{2x^2-4x-48}-\dfrac{5x+1}{x^2-36}$
36. $\dfrac{7}{2a-4}+\dfrac{9}{a-2}-\dfrac{6}{2-a}$
37. $\dfrac{x+2}{x^2-3x-28}-\dfrac{3x-7}{4x+16}-\dfrac{5-x}{8x-56}$
38. $\dfrac{a+b}{a^2-ab-2b^2}+\dfrac{3a-b}{a^2-4b^2}$
39. In electricity, the total resistance R_t of any parallel circuit is given by

$$\frac{1}{R_t}=\frac{I_1}{E_1}+\frac{I_2}{E_2}+\frac{I_3}{E_3}$$

Combine the expression in the right member.

[4–4]

Simplify each complex rational expression. Assume all denominators are nonzero.

40. $\dfrac{\frac{4}{a^2}}{\frac{2}{a}}$

41. $\dfrac{\frac{5}{x-3}}{\frac{4}{x}}$

42. $\dfrac{\frac{3x}{x-5}}{\frac{x}{x+2}}$

43. $\dfrac{7-\frac{3}{b-5}}{8+\frac{6}{b-5}}$

44. $\dfrac{x-y}{\frac{2}{x}+\frac{3}{y}}$

45. $\dfrac{(p-3)+\frac{2}{p-1}}{(p+1)-\frac{3}{p-1}}$

[4–5]

Perform the indicated divisions. Assume that no divisor is equal to zero.

46. $\dfrac{25a^7+15a^3+10a}{5a}$

47. $\dfrac{36a^5b^5c^3-18a^2b^3c^4+6a^2b^3c}{6a^2b^3c}$

48. $(3x^3+x+4)\div(x+1)$

49. $\dfrac{2x^4+x^3-3x^2+3x-1}{2x-1}$

[4–6]

Use synthetic division and the remainder theorem to evaluate each polynomial for the given value of the variable.

50. $P(x)=x^3-2x^2+8x-3;\ x=-2$

51. $P(y)=4y^3+2y^2-y+6;\ y=1$

52. $P(z)=-3z^4-z^2+5z-3;\ z=-1$

Use synthetic division to determine if the given number is a solution of the equation.

53. $x^3+2x^2+3x+18=0;\ -3$

54. $2y^3+4y^2+y-3=0;\ -1$

55. $4z^3-10z^2-z-6=0;\ 2$

[4–7]

Find the solution set of each of the following rational equations.

56. $6-\dfrac{10}{9a}=\dfrac{5}{12a}$

57. $\dfrac{4m-3}{15}+\dfrac{2-5m}{20}=1$

58. $\dfrac{4}{a+6}-\dfrac{9}{2a+12}=3$

59. $\dfrac{9}{4a^2-1}-\dfrac{6}{2a-1}=\dfrac{4}{2a+1}$

60. $\dfrac{8}{6n^2-13n-5}+\dfrac{6}{10n^2-25n}=0$

Solve the given equations for the indicated variable.

61. $\dfrac{2+m}{6}-\dfrac{n-3}{4}=\dfrac{p}{8}$ for p

62. $F=\dfrac{9}{5}C+32$ for C (Temperature conversion)

63. $\dfrac{P_1V_1}{T_1}=\dfrac{P_2V_2}{T_2}$ for V_1 (Physics formula)

64. In electricity, the total resistance R_t for two resistors connected in parallel is given by

$$R_t=\frac{R_1R_2}{R_1+R_2}$$

Solve for R_2.

[4–8]

Set up an equation to solve the following problems.

65. If Erin can do a job in 3 days and Sarah takes 6 days to do the same job, how long would it take the girls to do the job working together?

66. A train travels 120 miles in the same time that an automobile travels 80 miles. If the train travels 30 miles per hour faster than the automobile, what is the speed of each?

67. A boat travels 14 miles downstream in the same amount of time that it takes to travel 10 miles upstream. If the boat travels at 18 miles per hour in still water, what is the speed of the current?

68. The reciprocal of four times a number is equal to the reciprocal of twice the number added to 7. Find the number.

Chapter 4 cumulative test

Evaluate the following expressions when $x = 2$ and $y = -3$.

[1–5] 1. $x^2 - 2xy + y^2$

[1–5] 2. $3(2x - 1) - 2(3x + 2)$

[1–5] 3. $\dfrac{3x^2}{5y^2}$

[4–4] 4. $\dfrac{\dfrac{1}{x} - \dfrac{1}{y}}{\dfrac{1}{x} + \dfrac{1}{y}}$

[1–5] 5. $\dfrac{2}{3x} - \dfrac{4}{5y} + \dfrac{1}{xy}$

Perform the indicated operations and simplify. Assume all denominators are nonzero.

[1–4] 6. $3(2x - 1) + 4(x + 5) - (3x + 6)$

[3–3] 7. $(4ab)(-2a^2b)(3a^3b^3)$

[3–2] 8. $(x + 4)(x^2 - 2x + 5)$

[4–2] 9. $\dfrac{10xy}{9x^2y} \cdot \dfrac{27x^3y^2}{15x^2y^2}$

[4–2] 10. $\dfrac{x^2 + 5x - 24}{x^2 - 2x} \div \dfrac{x^2 - 9}{x^2 - 4x + 4}$

[4–3] 11. $\dfrac{2y - 3}{8} - \dfrac{4y + 1}{6}$

[3–2] 12. $(5x - 3)(2x + 9)$

[3–2] 13. $(4x - 5)^2$

[3–2] 14. $(3y - 5)(3y + 5)$

[4–5] 15. $(2x^3 + 3x^2 - 1) \div (x - 3)$

[4–2] 16. $\dfrac{6a^2 - a - 2}{a^2 + 4a - 21} \cdot \dfrac{2a^2 - a - 15}{8a^2 - 6a - 5}$

[4–3] 17. $\dfrac{4}{2x^2 - x - 3} - \dfrac{5}{2x^2 + 3x + 1}$

State the domain of the given rational expression.

[4–1] 18. $\dfrac{4y + 1}{2y - 3}$

[4–1] 19. $\dfrac{3x}{4x^2 - 1}$

[4–1] 20. $\dfrac{x - 5}{x^2 - 25}$

Find the solution set of each equation.

[2–1] 21. $4(y + 3) - 3(3y + 1) + (4y + 5) = 0$

[4–7] 22. $\dfrac{3}{4x} - 2 = \dfrac{5}{3x}$

[4–7] 23. $\dfrac{3}{y + 2} + \dfrac{2y + 1}{y^2 - 4} = \dfrac{-3}{y - 2}$

Simplify each complex rational expression. Assume all denominators are nonzero.

[4–4] 24. $\dfrac{\dfrac{1}{4} + \dfrac{2}{5}}{\dfrac{3}{10} - 1}$

[4–4] 25. $\dfrac{4 - \dfrac{3}{x + 2}}{5 + \dfrac{2}{x - 2}}$

Find the solution set of the following inequalities.

[2–5] 26. $5 - 3x \geq 8$

[2–5] 27. $3(4y - 1) < 2(5y + 7)$

[2–5] 28. $\dfrac{z + 1}{4} - \dfrac{z - 3}{6} > \dfrac{1}{3}$

[2–5] 29. $\dfrac{3}{4}(x + 1) - 2(x - 3) \leq \dfrac{2}{3}(x - 2)$

Reduce the following rational expressions to lowest terms. Assume all denominators are nonzero.

[4–1] 30. $\dfrac{36a^2b^3}{-24ab^5}$

[4–1] 31. $\dfrac{p^2 + p - 12}{p^2 - 9}$

[4–1] 32. $\dfrac{12 - 6y}{4y - 8}$

[4–7] 33. Find the solution set of the rational equation
$$\frac{4}{x} - 3 = 6 + \frac{5}{x}.$$

[4–8] 34. The sum of twice a number and six times its reciprocal is 13. Find the number.

[4–6] 35. Using synthetic division, find $P(-2)$ when $P(x) = x^4 - x^3 + 2x^2 - 5$.

Exercise 3-8

Answers to odd-numbered problems

1. $(m-7)(m+7)$ **3.** $(x+5)(x+1)$ **5.** $(7a+1)(a+5)$ **7.** $(2a+3)(a+6)$ **9.** $(ab+4)(ab-2)$ **11.** $(3a+b)(9a^2-3ab+b^2)$ **13.** $5(3x+y)(5x^2+a)$ **15.** $10(x-y)^2$ **17.** $4(m-2n)(m+2n)$ **19.** $(a-b-2x-y)(a-b+2x+y)$ **21.** $3(x^2-3y)(x^4+3x^2y+9y^2)$ **23.** $2xy^2(6x^2-9x+8y^2)$ **25.** $(2x+3y)(2x-3y)$ **27.** $(x+2y)(3a-b)$ **29.** $(3a^3-bc)(9a^6+3a^3bc+b^2c^2)$ **31.** $(5a+3)(a-7)$ **33.** $(a-1)(a+1)(a-2)(a+2)$ **35.** $(2a+3b)(2a-5b)$ **37.** $(y-2)(y+2)(y^2+4)$ **39.** $2(a+2)(2a+1)$ **41.** $(x+y-9)(x+y+1)$ **43.** $(2a-1)(3a+5)$ **45.** $4ab(x+3y)(1-2ab)$ **47.** $(2a-5b)^2$ **49.** $5x(4x^2+1)(2x+1)(2x-1)$ **51.** $3ab(a^2-3b^2)^2$ **53.** $(3a-x-5y)(3a+x+5y)$ **55.** $(3x-13)(x+7)$ **57.** $3x^2(xy^3+3z^2)(x^2y^6-3xy^3z^2+9z^4)$ **59.** $(3-x)(3+x)(a-3)^2$

Review exercises

1. $\{7\}$ **2.** $\left\{-\frac{3}{2}\right\}$ **3.** $\left\{\frac{4}{5}\right\}$ **4.** $\{0\}$ **5.** $\left\{\frac{6}{5}\right\}$ **6.** $\{2\}$

Chapter 3 review

1. $-15x^5$ **2.** $6a^3b^5$ **3.** $8a^9b^{12}c^3$ **4.** $x^8y^9z^2$ **5.** $17a^7$ **6.** $4b^{10}-27b^8$ **7.** $10a^4b-15a^3b^2+20a^2b^3$ **8.** $2a^2+ab-b^2$ **9.** y^2-49 **10.** $4a^2+4ab+b^2$ **11.** $a^3-2a^2b-ab^2+2b^3$ **12.** $2x^2+16x-17$ **13.** $\frac{b^3}{4a^3}$ **14.** $\frac{6}{x^9}$ **15.** $\frac{9y^6}{x^4}$ **16.** $\frac{64x^9}{y^{15}}$ **17.** $\frac{a^9}{b^3}$ **18.** $\frac{a^3c^7}{2b}$ **19.** $\frac{4}{y^5}$ **20.** $6x^2(2x-3)$ **21.** $4a^2b(3a^2-b^2+6ab)$ **22.** $(x-2y)(a+b)$ **23.** $5x(2x+1)(x-3z)$ **24.** $(2x-y)(3a-b)$ **25.** $(2y+x)(4a+3b)$ **26.** $(2x+y)(2a+3b)$ **27.** $(a+3b)(x-2y)$ **28.** $(2x-y)(3a-b)$ **29.** $(a+3b)(2x+y)$ **30.** $(a+2)(a+12)$ **31.** $(b-7)(b-2)$ **32.** $2a(a-5)(a+1)$ **33.** $x(x-3)(x+2)$ **34.** $(xy+6)(xy+4)$ **35.** $(ab-10)(ab+2)$ **36.** $(x+3y-1)(x+3y+2)$ **37.** $(x+2y-7)^2$ **38.** $(a-2b)(a+b)$ **39.** $(x+2y)(x+3y)$ **40.** $(2a-3)(a-2)$ **41.** $(4x-5)(2x-1)$ **42.** $(2y+1)(3y-4)$ **43.** $(3a-1)(a+1)$ **44.** $(4x-1)(x+3)$ **45.** $(2x+1)(2x+1)=(2x+1)^2$ **46.** $(6a+1)(4a+3)$ **47.** $(4x-3)(2x-3)$ **48.** $(2x+3)(x+6)$ **49.** $3(2x+1)(x+8)$ **50.** $(-2x-1)(x-6)$ or $(2x+1)(-x+6)$ or $-(2x+1)(x-6)$ **51.** $(x+9)(x-9)$ **52.** $4(a+3b)(a-3b)$ **53.** $3(a+3b)(a-3b)$ **54.** $(y^2+9)(y+3)(y-3)$ **55.** $(a+2b)(x+y)(x-y)$ **56.** $(x-3z)(2a+3b)(2a-3b)$ **57.** $(a+2b)(a^2-2ab+4b^2)$ **58.** $(3x-y)(9x^2+3xy+y^2)$ **59.** $8(2a+b)(4a^2-2ab+b^2)$ **60.** $x^2(3x+y)(9x^2-3xy+y^2)$ **61.** $2(x-2b)(x^2+2bx+4b^2)$ **62.** $(x^4y^5-z^3)(x^8y^{10}+x^4y^5z^3+z^6)$ **63.** $(3x+4)^2$ **64.** $(a^2-b^3)(a^4+a^2b^3+b^6)$ **65.** $3a(a+5)(a-5)$ **66.** $(a+6)^2(3+x)(3-x)$ **67.** $(6a-1)(a+3)$ **68.** $(x-2y)(4m+3n)$ **69.** $5x(2x+y)(4x^2-2xy+y^2)$ **70.** $ab^3(a-2b)^2$ **71.** $(x+4y)(x-2y)$ **72.** $(3a+2)(5a-2)$

Chapter 3 cumulative test

1. false **2.** false **3.** true **4.** 2 **5.** undefined **6.** 4 **7.** 0 **8.** 81 **9.** 37 **10.** -34 **11.** $\{x|x \geq 7\}$ **12.** $\{5\}$ **13.** $\left\{-3,\frac{1}{3}\right\}$ **14.** $\left\{x|-\frac{1}{3}<x<\frac{17}{3}\right\}$ **15.** $\left\{x|x<\frac{-11}{6} \text{ or } x>\frac{1}{6}\right\}$ **16.** $\left\{x|-\frac{3}{2}<x<\frac{7}{2}\right\}$ **17.** $\left\{x|-5 \leq x \leq \frac{-3}{2}\right\}$ **18.** let x = the number; $\{x|x>9\}$ **19.** \$4,000 **20.** $6a^5b^5$ **21.** $16x^{12}y^{16}$ **22.** $\frac{3a^3b^5}{2c^6}$ **23.** $\frac{y^4}{x^6}$ **24.** $\frac{y^9}{27x^9}$ **25.** $4x^2-2x-3$ **26.** $\frac{a^9c^3}{27b^6}$ **27.** $7ab-a-5b$ **28.** $9a^2-6ab+b^2$ **29.** $8a^{18}b^{11}$ **30.** $12x^6y^3-18x^6y^4+24x^4y^5$ **31.** $(x+6)(x-6)$ **32.** $(x+3y)(x^2-3xy+9y^2)$ **33.** $(2a-3)(a-6)$ **34.** $(y+6)(y+1)$ **35.** $(xy+2)(xy-4)$ **36.** $(7x+1)(x-5)$ **37.** $(2a+3b)(x-3y)$ **38.** $5x(2a-b)(3x+1)$ **39.** $(2x-5y)^2$ **40.** $3(a+2b)(a-2b)$

Chapter 4

Exercise 4-1

Answers to odd-numbered problems

1. domain $=\{x|x \in R, x \neq 4\}$ **3.** domain $=\{x|x \in R, x \neq -1\}$ **5.** domain $=\left\{z|z \in R, z \neq \frac{3}{2}\right\}$ **7.** domain $=\left\{x|x \in R, x \neq -\frac{4}{7}\right\}$ **9.** domain $=\{x|x \in R, x \neq 0,4\}$ **11.** domain $=\{x|x \in R, x \neq -4\}$ **13.** domain $=\left\{y|y \in R, y \neq -\frac{5}{2},\frac{5}{2}\right\}$ **15.** domain $=\left\{x|x \in R, x \neq -\frac{1}{2},3\right\}$ **17.** domain $=\{x|x \in R\}$ **19.** $\frac{5}{3x}\ (x \neq 0)$ **21.** $\frac{p^2}{q^2}\ (p \neq 0, q \neq 0)$ **23.** $-\frac{ab^3}{c^4}\ (a \neq 0, b \neq 0, c \neq 0)$ **25.** $\frac{3n^2p^2}{4m^3}\ (m \neq 0, n \neq 0, p \neq 0)$ **27.** $\frac{3}{7}\ (x \neq 2)$ **29.** $\frac{3}{a-2}\ (a \neq 0,2)$ **31.** $2(x-1)\ (x \neq 0)$ **33.** $\frac{4(y+1)}{3}\ (y \neq 1)$ **35.** $-5\ (x \neq y)$ **37.** $\frac{a-3}{4}\ (a \neq -3)$ **39.** $2y-1\ \left(y \neq -\frac{1}{2}\right)$ **41.** $x^2-2x+4\ (x \neq -2)$ **43.** $\frac{-3}{2(y^2+xy+x^2)}\ (x \neq y)$ **45.** $\frac{y-7}{y+7}\ (y \neq -7)$ **47.** $\frac{m-6}{m-3}\ (m \neq -2,3)$ **49.** $\frac{y-8}{y-5}\ (y \neq -4,5)$ **51.** $\frac{a-1}{a+1}\ \left(a \neq -1,\frac{1}{2}\right)$ **53.** $\frac{2x-3}{4x+1}\ \left(x \neq -3,-\frac{1}{4}\right)$ **55.** $\frac{3m+2}{2m+1}\ \left(x \neq -\frac{3}{4},-\frac{1}{2}\right)$ **57.** $\frac{2(a+2)}{3a+1}\ \left(a \neq -\frac{1}{3},\frac{3}{2}\right)$ **59.** $\frac{5(y+1)}{4(y+3)}\ (y \neq -3,3)$ **61.** $\frac{-(a+6)}{3+a}\ \left(a \neq -3,\frac{2}{3}\right)$ **63.** $\frac{p+4q}{p+2q}\ (p \neq -3q,-2q)$

Solutions to trial exercise problems

5. Since $2z - 3 = 0$ when $z = \frac{3}{2}$, the domain is $\left\{z \mid z \in R, z \neq \frac{3}{2}\right\}$.

10. Since $3m^2 + 6m = 3m(m + 2)$ and $3m = 0$ when $m = 0$, $m + 2 = 0$ when $m = -2$, the domain is $\{m \mid m \in R, m \neq 0, -2\}$.

13. Since $4y^2 - 25 = (2y + 5)(2y - 5)$ and $2y + 5 = 0$ when $y = -\frac{5}{2}$, $2y - 5 = 0$ when $y = \frac{5}{2}$, the domain is $\left\{y \mid y \in R, y \neq -\frac{5}{2}, \frac{5}{2}\right\}$. 17. Since $x^2 + 4 \neq 0$ for any value of x, the domain is $\{x \mid x \in R\}$.

30. $\dfrac{-36x}{42x^3 + 24x} = \dfrac{-36x}{6x(7x^2 + 4)} = \dfrac{6x \cdot (-6)}{6x(7x^2 + 4)} = \dfrac{-6}{7x^2 + 4}\ (x \neq 0)$

35. $\dfrac{5x - 5y}{y - x} = \dfrac{5(x - y)}{-(x - y)} = \dfrac{5}{-1} = -5\ (x \neq y)$

41. $\dfrac{x^3 + 8}{x + 2} = \dfrac{(x + 2)(x^2 - 2x + 4)}{x + 2} = x^2 - 2x + 4\ (x \neq -2)$

53. $\dfrac{2x^2 + 3x - 9}{4x^2 + 13x + 3} = \dfrac{(2x - 3)(x + 3)}{(x + 3)(4x + 1)} = \dfrac{2x - 3}{4x + 1}$ $\left(x \neq -3, -\frac{1}{4}\right)$ 63. $\dfrac{p^2 + 7pq + 12q^2}{p^2 + 5pq + 6q^2} = \dfrac{(p + 3q)(p + 4q)}{(p + 2q)(p + 3q)}$ $= \dfrac{p + 4q}{p + 2q}\ (p \neq -3q, -2q)$

Review exercises

1. $-\frac{1}{12}$ 2. 1 3. $\frac{13}{8}$ or $1\frac{5}{8}$ 4. $\{2\}$ 5. $\left\{\frac{9}{2}\right\}$
6. $x = \dfrac{2a - 5}{2}$

Exercise 4–2

Answers to odd-numbered problems

1. $\frac{8}{3x}\ (x \neq 0)$ 3. $\frac{1}{2x^2y^2}\ (x,y \neq 0)$ 5. $\frac{a^2}{7bc^3}\ (a,b,c \neq 0)$
7. $2ab^2\ (a,b \neq 0)$ 9. $\frac{4a - 8}{15a - 60}\ (a \neq 4, -2)$ 11. $\frac{a - 6}{6}\ (a \neq -3)$
13. $\frac{-4p^2 - 5p - 1}{p - 1}\left(p \neq -1, \frac{1}{4}, 1\right)$ 15. $\frac{x - 5}{x - 3}\ (x \neq 1,2,3,4)$
17. $\frac{3a^2 - 5a - 2}{3a^2 - 4a + 1}\left(a \neq -2, -\frac{3}{2}, \frac{1}{3}, 1\right)$
19. $\frac{x^2 - 2x + 4}{2x + 7}\left(x \neq -2, -\frac{7}{2}, 2\right)$ 21. $\frac{-1}{x + 2y}\ (x \neq -2y, -y, y)$
23. $\frac{a^2}{b^3}$ 25. $\frac{1}{r^2 - 5r + 4}$ 27. $\frac{1}{3x + 12}$ 29. $\frac{x^3 + x^2 - x - 1}{4}$
31. $\frac{x^3 - 125}{x + 5}$ 33. $\frac{-x^2 - 3x + 40}{x^2 + 4x - 21}$ 35. $\frac{x + 4}{x - 4}$ 37. $\frac{a - b}{2}$
39. $-(2m^2 + 5mn + 3n^2)$ 41. $\frac{a - 6}{a + 3}$ 43. $\frac{x^2 - 9}{x^2 - 5x + 4}$
45. $-(9y^2 + 6xy + 4x^2)(x^2 - xy + y^2)$ 47. $\frac{b^2 + b - 2}{b^2 - b - 2}$
49. $\frac{z^2 - 2wz + w^2}{z^2 + 2wz + w^2}$

Solutions to trial exercise problems

9. $\dfrac{4a + 8}{3a - 12} \cdot \dfrac{a - 2}{5a + 10} = \dfrac{4(a + 2) \cdot (a - 2)}{3(a - 4) \cdot 5(a + 2)} = \dfrac{4(a - 2)}{15(a - 4)}$ $= \dfrac{4a - 8}{15a - 60}\ (a \neq -2, a \neq 4)$ 17. $\dfrac{2a^2 - a - 6}{3a^2 - 4a + 1} \cdot \dfrac{3a^2 + 7a + 2}{2a^2 + 7a + 6}$ $= \dfrac{(2a + 3)(a - 2)}{(3a - 1)(a - 1)} \cdot \dfrac{(3a + 1)(a + 2)}{(2a + 3)(a + 2)} = \dfrac{(a - 2)(3a + 1)}{(3a - 1)(a - 1)}$ $= \dfrac{3a^2 - 5a - 2}{3a^2 - 4a + 1}\left(a \neq -2, -\frac{3}{2}, \frac{1}{3}, 1\right)$

25. $\dfrac{r + 4}{r^2 - 1} \div \dfrac{r^2 - 16}{r + 1} = \dfrac{(r + 4) \cdot (r + 1)}{(r^2 - 1) \cdot (r^2 - 16)}$ $= \dfrac{(r + 4)(r + 1)}{(r + 1)(r - 1)(r + 4)(r - 4)} = \dfrac{1}{(r - 4)(r - 1)}$ $= \dfrac{1}{r^2 - 5r + 4}$ 33. $\dfrac{56 - x - x^2}{x^2 - 6x - 7} \div \dfrac{x^2 + 4x - 21}{x^2 - 4x - 5}$ $= \dfrac{-(x^2 + x - 56) \cdot (x^2 - 4x - 5)}{(x^2 - 6x - 7) \cdot (x^2 + 4x - 21)}$ $= \dfrac{-(x - 7)(x + 8) \cdot (x - 5)(x + 1)}{(x - 7)(x + 1) \cdot (x + 7)(x - 3)} = \dfrac{-(x + 8)(x - 5)}{(x + 7)(x - 3)}$ $= \dfrac{-(x^2 + 3x - 40)}{x^2 + 4x - 21} = \dfrac{-x^2 - 3x + 40}{x^2 + 4x - 21}$

39. $\dfrac{n^2 - m^2}{2m - 3n} \div \dfrac{m - n}{4m^2 - 9n^2} = \dfrac{(n^2 - m^2) \cdot (4m^2 - 9n^2)}{(2m - 3n) \cdot (m - n)}$ $= \dfrac{-(m - n)(m + n) \cdot (2m + 3n)(2m - 3n)}{(2m - 3n) \cdot (m - n)}$ $= -(m + n)(2m + 3n) = -(2m^2 + 5mn + 3n^2)$

41. $\dfrac{a^2 - 8a + 15}{a^2 + 7a + 6} \cdot \dfrac{a^2 - 6a - 7}{a^2 - 9} \div \dfrac{a^2 - 12a + 35}{a^2 - 36}$ $= \dfrac{(a^2 - 8a + 15) \cdot (a^2 - 6a - 7) \cdot (a^2 - 36)}{(a^2 + 7a + 6) \cdot (a^2 - 9) \cdot (a^2 - 12a + 35)}$ $= \dfrac{(a - 3)(a - 5) \cdot (a - 7)(a + 1) \cdot (a + 6)(a - 6)}{(a + 6)(a + 1) \cdot (a + 3)(a - 3) \cdot (a - 7)(a - 5)} = \dfrac{a - 6}{a + 3}$

47. $\dfrac{ab - a + 3b - 3}{ab + a - 4b - 4} \div \dfrac{ab - 2a + 3b - 6}{ab + 2a - 4b - 8}$ $= \dfrac{(ab - a + 3b - 3) \cdot (ab + 2a - 4b - 8)}{(ab + a - 4b - 4) \cdot (ab - 2a + 3b - 6)}$ $= \dfrac{(a + 3)(b - 1) \cdot (a - 4)(b + 2)}{(a - 4)(b + 1)\,(a + 3)(b - 2)} = \dfrac{(b - 1)(b + 2)}{(b + 1)(b - 2)}$ $= \dfrac{b^2 + b - 2}{b^2 - b - 2}$

Review exercises

1. $\frac{43}{24}$ or $1\frac{19}{24}$ 2. $\frac{1}{3}$ 3. $\{y \mid y \leq 2\} = (-\infty, 2]$
4. $\{y \mid -3 \leq y < 1\} = [-3,1]$ 5. $8x^5 - 4x^4 + 24x^3$
6. $4y^2 - 15y - 4$

Exercise 4–3

Answers to odd-numbered problems

1. $70x$ 3. $80k^2$ 5. $288a^3b^2$ 7. $30x^2y^4$ 9. $4a(a - 2)$
11. $6x(x - 7)$ 13. $3(p + 3)^2(p - 4)$ 15. $10(a + 5)(a - 5)$
17. $(q + 7)(q - 7)(q - 2)$ 19. $5(a + b)(a - b)$
21. $(2a + 1)(3a - 1)(a - 7)(a + 7)$ 23. $\frac{12x^2}{21x^3}$ 25. $\frac{15x^3y}{24x^2y^2}$

27. $\dfrac{4p^2 - 12p}{p - 3}$ **29.** $\dfrac{p^2 - 5p + 6}{p^2 - 4}$ **31.** $\dfrac{16x^3 + 24x^2 + 36x}{8x^3 - 27}$
33. $\dfrac{2n^2 - 11n + 5}{n^2 + 2n - 35}$ **35.** $\dfrac{2y^2 - y - 10}{4y^2 + 7y - 2}$ **37.** $\dfrac{15m^2 + 12m}{25m^2 - 16}$
39. $\dfrac{9}{a - 9}$ **41.** $\dfrac{13}{q}$ **43.** $\dfrac{-3y}{y + 4}$ **45.** 5 **47.** 1 **49.** $\dfrac{1}{x + 3}$
51. 1 **53.** $\dfrac{47}{12x}$ **55.** $\dfrac{5z - 2}{z^2}$ **57.** $\dfrac{18x + 5}{9x^2}$ **59.** $\dfrac{6y - 4x}{9x^2y^2}$
61. $\dfrac{14a^2 - 2a}{(3a + 5)(2a - 3)}$ **63.** $\dfrac{129y - 122}{10(y - 2)(y + 2)}$ **65.** $\dfrac{2x - 4}{x - 9}$
67. $\dfrac{16x - 61}{(x - 6)(x + 1)(x - 1)}$ **69.** $\dfrac{5x^2 + 3xy + 6y^2}{(x - 3y)(x + y)^2}$
71. $\dfrac{2a^2 - 16a - 25}{(2a - 1)(4a + 3)(a + 5)}$ **73.** $\dfrac{20a^2 - 25a + 1}{5a - 2}$
75. $\dfrac{17p^2 - 20p - 75}{8p(p - 5)(p + 3)}$ **77.** $\dfrac{3y^2 - y + 15}{(y + 3)(y - 2)(y + 2)}$
79. $\dfrac{5b^2 + 7b - 30}{5(b + 2)(b - 2)}$ **81.** $\dfrac{3x^2 - 15xy + 14y^2}{(x - 6y)^2(x + 2y)}$
83. $\dfrac{15x^2 - 9x - 9}{(4x - 3)(2x - 5)(3x + 1)}$
85. $\dfrac{49m^2 - 20mn + 3n^2}{(8m - n)(m + 2n)(5m - 4n)}$ **87.** $\dfrac{qrs + prs + pqs + pqr}{pqrs}$
89. $\dfrac{f_1 + f_2 - d}{f_1f_2}$ **91.** $\dfrac{(n - 1)(R_2 + R_1)}{R_1R_2}$

Solutions to trial exercise problems

13. $p^2 - p - 12 = (p - 4)(p + 3)$; $p^2 + 6p + 9 = (p + 3)^2$; $3p - 12 = 3(p - 4)$. The LCM is $3(p + 3)^2(p - 4)$.
18. $2y + 10 = 2(y + 5)$; $25 - y^2 = -(y + 5)(y - 5)$; $y^2 - 10y + 25 = (y - 5)^2$. The LCM is $2(y + 5)(y - 5)^2$.
21. $2a^2 - 13a - 7 = (2a + 1)(a - 7)$; $6a^2 + a - 1 = (3a - 1)(2a + 1)$; $a^2 - 49 = (a + 7)(a - 7)$. The LCM is $(2a + 1)(3a - 1)(a - 7)(a + 7)$.
31. Since $8x^3 - 27 = (2x - 3)(4x^2 + 6x + 9)$, then $\dfrac{4x}{2x - 3} = \dfrac{4x \cdot (4x^2 + 6x + 9)}{(2x - 3) \cdot (4x^2 + 6x + 9)} = \dfrac{16x^3 + 24x^2 + 36x}{8x^3 - 27}$. **37.** Since $25m^2 - 16 = (5m + 4)(5m - 4)$, then $-\dfrac{3m}{4 - 5m} = -\dfrac{3m}{-(5m - 4)} = \dfrac{3m}{5m - 4} = \dfrac{3m \cdot (5m + 4)}{(5m - 4)(5m + 4)} = \dfrac{15m^2 + 12m}{25m^2 - 16}$.
53. $\dfrac{8}{3x} + \dfrac{5}{4x} = \dfrac{8(4)}{3x(4)} + \dfrac{5(3)}{4x(3)} = \dfrac{32}{12x} + \dfrac{15}{12x} = \dfrac{32 + 15}{12x} = \dfrac{47}{12x}$
65. $\dfrac{x + 2}{x - 9} - \dfrac{x - 6}{9 - x} = \dfrac{x + 2}{x - 9} - \dfrac{x - 6}{-(x - 9)}$
$= \dfrac{x + 2}{x - 9} + \dfrac{x - 6}{x - 9} = \dfrac{(x + 2) + (x - 6)}{x - 9} = \dfrac{x + 2 + x - 6}{x - 9}$
$= \dfrac{2x - 4}{x - 9}$ **70.** $\dfrac{8q}{4q^2 - 9p^2} + \dfrac{5q}{4q^2 - 12pq + 9p^2}$
$= \dfrac{8q}{(2q + 3p)(2q - 3p)} + \dfrac{5q}{(2q - 3p)^2} = \dfrac{8q(2q - 3p)}{(2q + 3p)(2q - 3p)^2}$
$+ \dfrac{5q(2q + 3p)}{(2q - 3p)^2(2q + 3p)} = \dfrac{(16q^2 - 24pq) + (10q^2 + 15pq)}{(2q + 3p)(2q - 3p)^2}$
$= \dfrac{26q^2 - 9pq}{(2q + 3p)(2q - 3p)^2}$

73. $(4a - 3) - \dfrac{2a + 5}{5a - 2} = \dfrac{4a - 3}{1} - \dfrac{2a + 5}{5a - 2}$
$= \dfrac{(4a - 3) \cdot (5a - 2) - 1 \cdot (2a + 5)}{1 \cdot (5a - 2)}$
$= \dfrac{20a^2 - 23a + 6 - 2a - 5}{5a - 2} = \dfrac{20a^2 - 25a + 1}{5a - 2}$
82. $\dfrac{a - b}{a^2 - 3ab - 4b^2} + \dfrac{2b - 5a}{a^2 - 16b^2} = \dfrac{a - b}{(a - 4b)(a + b)}$
$+ \dfrac{2b - 5a}{(a + 4b)(a - 4b)} = \dfrac{(a - b) \cdot (a + 4b)}{(a - 4b)(a + b)(a + 4b)}$
$+ \dfrac{(2b - 5a) \cdot (a + b)}{(a + 4b)(a - 4b)(a + b)}$
$= \dfrac{(a^2 + 3ab - 4b^2) + (2b^2 - 3ab - 5a^2)}{(a + 4b)(a - 4b)(a + b)}$
$= \dfrac{a^2 + 3ab - 4b^2 + 2b^2 - 3ab - 5a^2}{(a + 4b)(a - 4b)(a + b)}$
$= \dfrac{-4a^2 - 2b^2}{(a + 4b)(a - 4b)(a + b)}$

Review exercises

1. $(2x^2 + 3)(4x - 1)$ **2.** $(2x + 1)(2x - 7)$
3. $(3y + 7)(3y - 7)$ **4.** 12 **5.** $24k$

Exercise 4–4

Answers to odd-numbered problems

1. $\dfrac{21}{20}$ **3.** $\dfrac{11}{3}$ **5.** $\dfrac{12}{11}$ **7.** $\dfrac{7}{8m}$ **9.** $\dfrac{-4y}{3y + 3}$ **11.** $\dfrac{x - 3}{x + 7}$
13. $\dfrac{ab}{b - a}$ **15.** $\dfrac{x}{x - 2y}$ **17.** $\dfrac{20}{27}$ **19.** $\dfrac{5}{7y}$ **21.** $\dfrac{35 - 7a}{4a + 36}$
23. $\dfrac{8x - 4y}{3}$ **25.** $\dfrac{4x^2 + 5x - 9}{5x^2 + 16x + 3}$ **27.** $\dfrac{xy^2 + x^2}{y^2 - xy^2}$
29. $\dfrac{mn(m + n)}{n - m}$ **31.** $\dfrac{5q + 4p^2}{p^2q(p - q)}$ **33.** $\dfrac{a^2 + 9a + 23}{a^2 + 7a + 7}$
35. $\dfrac{4y^3 + 17y^2 - 18y - 15}{4y^3 + 25y^2 + 31y - 39}$ **37.** $\dfrac{t^2 - 3t - 4}{t^2 + 3t - 18}$ **39.** $\dfrac{-3}{2x + 14}$
41. $\dfrac{5b^2 + 24b - 5}{4b - 22}$ **43.** $\dfrac{1}{2}$ **45.** $\dfrac{3c + 4a - 2b}{5}$ **47.** $\dfrac{y + x}{y - x}$
49. $\dfrac{y}{y + x}$ **51.** $\dfrac{q^2 + p^2}{p^2}$ **53.** $\dfrac{pq}{q - p}$ **55.** $\dfrac{V_sC - it}{RC}$
57. $CP = \dfrac{T_1}{T_2 - T_1}$ **59.** $r = \dfrac{2dr_1r_2}{dr_2 + dr_1} = \dfrac{2r_1r_2}{r_2 + r_1}$

Solutions to trial exercise problems

25. $\dfrac{4 - \dfrac{3}{x + 3}}{5 + \dfrac{6}{x - 1}} = \dfrac{4(x + 3)(x - 1) - \dfrac{3}{x + 3} \cdot (x + 3)(x - 1)}{5(x + 3)(x - 1) + \dfrac{6}{x - 1} \cdot (x + 3)(x - 1)}$
$= \dfrac{4(x^2 + 2x - 3) - 3(x - 1)}{5(x^2 + 2x - 3) + 6(x + 3)}$
$= \dfrac{4x^2 + 8x - 12 - 3x + 3}{5x^2 + 10x - 15 + 6x + 18}$
$= \dfrac{4x^2 + 5x - 9}{5x^2 + 16x + 3}$
28. $\dfrac{\dfrac{1}{a} - \dfrac{1}{b}}{\dfrac{1}{a^2} + \dfrac{1}{b^2}} = \dfrac{\dfrac{1}{a} \cdot a^2b^2 - \dfrac{1}{b} \cdot a^2b^2}{\dfrac{1}{a^2} \cdot a^2b^2 + \dfrac{1}{b^2} \cdot a^2b^2} = \dfrac{ab^2 - a^2b}{b^2 + a^2}$

33. $$\frac{(a+5)+\frac{3}{a+4}}{(a+3)-\frac{5}{a+4}} = \frac{(a+5)(a+4)+\frac{3}{a+4}(a+4)}{(a+3)(a+4)-\frac{5}{a+4}(a+4)}$$
$$= \frac{a^2+9a+20+3}{a^2+7a+12-5} = \frac{a^2+9a+23}{a^2+7a+7}$$

39. $$\frac{\frac{3}{x^2-x-6}}{\frac{2}{x+2}-\frac{4}{x-3}} = \frac{\frac{3}{(x-3)(x+2)}}{\frac{2}{x+2}-\frac{4}{x-3}}$$
$$= \frac{\frac{3}{(x-3)(x+2)}\cdot(x-3)(x+2)}{\frac{2}{x+2}(x-3)(x+2)-\frac{4}{x-3}(x-3)(x+2)}$$
$$= \frac{3}{2(x-3)-4(x+2)} = \frac{3}{2x-6-4x-8}$$
$$= \frac{3}{-2x-14} = \frac{3}{-(2x+14)} = \frac{-3}{2x+14}$$

Review exercises

1. $\left\{-\frac{14}{3}\right\}$ 2. $\{-2,8\}$ 3. $\{2\}$ 4. $\{x \mid -2 < x < 6\} = (-2,6)$
5. $\{x \mid x \le -4 \text{ or } x \ge -1\} = (-\infty,-4] \cup [-1,\infty)$
6. $2a^3 + 8a^2 - a - 4$

Exercise 4-5

Answers to odd-numbered problems

1. $5x^2 - 3x + 2$ 3. $-x^3 + 2x^2 - 3$ 5. $c - 1$
7. $10xy^2 + 7 - 6y^2$ 9. $3a^6b - 5a^4b^4 + 7a^3b$ 11. $2a + 3c$
13. $a - 4$ 15. $y + 2 + \frac{1}{y+5}$ 17. $x - 4 - \frac{2}{x-3}$
19. $a^2 - 2a + 4$ 21. $2y^2 - 3y + 1 + \frac{2}{y+1}$ 23. $x^2 + 3x - 4$
25. $2a^2 + 3a - 4 + \frac{5}{2a+1}$ 27. $3y^2 - y + 4 + \frac{2}{3y+1}$
29. $a^2 - 3a + 9$ 31. $3x^3 - 5x^2 + 5x - 4 + \frac{3}{x+1}$
33. $3a - 1$ 35. $y^2 + y - 1$ 37. $x^2 + 3x + 2 - \frac{4}{x^2+x-4}$
39. $3a^2 - 2a - 4$ 41. $y^2 + 3 + \frac{2}{2y^2-3y+2}$ 43. 2, it is the same. 45. $2x - 3$ 47. $3x - 2$

Solutions to trial exercise problems

10. $\frac{x(y-2)-z(y-2)}{y-2} = \frac{x(y-2)}{y-2} - \frac{z(y-2)}{y-2} = x - z$
16. $(2x - 5 + x^2) \div (x + 4) = (x^2 + 2x - 5) \div (x + 4)$

$$\begin{array}{r} x - 2 \\ x+4\overline{)x^2+2x-5} \\ \underline{x^2+4x} \\ -2x-5 \\ \underline{-2x-8} \\ 3 \end{array}$$

Answer: $x - 2 + \frac{3}{x+4}$

27. $\frac{9y^3+11y+6}{3y+1}$

$$\begin{array}{r} 3y^2 - y + 4 \\ 3y+1\overline{)9y^3+0y^2+11y+6} \\ \underline{9y^3+3y^2} \\ -3y^2+11y \\ \underline{-3y^2-y} \\ 12y+6 \\ \underline{12y+4} \\ 2 \end{array}$$

Answer: $3y^2 - y + 4 + \frac{2}{3y+1}$

32. $\frac{2x^3+5x^2+5x+3}{x^2+x+1}$

$$\begin{array}{r} 2x + 3 \\ x^2+x+1\overline{)2x^3+5x^2+5x+3} \\ \underline{2x^3+2x^2+2x} \\ 3x^2+3x+3 \\ \underline{3x^2+3x+3} \\ 0 \end{array}$$

Answer: $2x + 3$

38. $\frac{2x^4-x^3+5x^2-x+3}{x^2+1}$

$$\begin{array}{r} 2x^2 - x + 3 \\ x^2+0x+1\overline{)2x^4-x^3+5x^2-x+3} \\ \underline{2x^4+0x^3+2x^2} \\ -x^3+3x^2-x \\ \underline{-x^3-0x^2-x} \\ 3x^2+0x+3 \\ \underline{3x^2+0x+3} \\ 0 \end{array}$$

Answer: $2x^2 - x + 3$

Review exercises

1. x 2. x^3 3. $\frac{8}{a^{12}}$ 4. $\frac{3}{10}$ 5. $\frac{-x^2+4x+2}{(x+3)(x-3)}$ 6. $\frac{2x-1}{2x}$
7. $\left\{\frac{19}{3}\right\}$

Exercise 4-6

Answers to odd-numbered problems

1. $a + 2$ 3. $a + 5$ 5. $x + 7 + \frac{23}{x-2}$ 7. $3a - 1$
9. $a^2 + 3a + 2 - \frac{2}{a-1}$ 11. $x^2 + x + 1$
13. $y^2 + 5y + 10 + \frac{16}{y-2}$ 15. $2a^2 + a - 3 + \frac{2}{a-5}$
17. $x^3 - 2x^2 - x + 3 - \frac{1}{x-3}$ 19. $3y^3 + 2y^2 - 4y - 1 + \frac{3}{y-3}$
21. $P(1) = -1$ 23. $P(-3) = 319$ 25. $P(0) = 1$
27. $P(2) = 145$ 29. $P\left(\frac{1}{2}\right) = -\frac{3}{2}$ 31. $P(-3) = 4$; $x + 3$ is not a factor 33. $P(2) = 0$; $x - 2$ is a factor
35. $P(-1) = 25$; $x + 1$ is not a factor 37. $P(-4) = 0$; $x + 4$ is a factor 39. $P(-1) = -6$; $x + 1$ is not a factor
41. $\left\{-8,\frac{2}{3},1\right\}$ 43. $\left\{-6,\frac{1}{2},5\right\}$ 45. $\{-1,1\}$
47. $P(x) = x^3 - 5x^2 + 2x + 8$ 49. $P(x) = x^3 - 3x^2 - 4x$

51. $P(x) = x^4 - 65x^2 + 64$ **53.** $P(x) = x^4 - 8x^3 - 13x^2 + 200x - 300$ **55.** $P(x) = x^4 - 6x^3 + x^2 + 24x + 16$ **57.** $P(x) = x^4 + 7x^3 + 10x$ **59.** 2 of multiplicity 2; -2 of multiplicity 2; 3 of multiplicity 3; -4 of multiplicity 2 **61.** 0 of multiplicity 2; -4 of multiplicity 4; 3 of multiplicity 2; 6 **63.** 25

65. Since the remainder is 0, $\frac{3}{2}$ is a solution.

Solutions to trial exercise problems

8. $\dfrac{5 - 3x + 2x^2}{x + 2} = \dfrac{2x^2 - 3x + 5}{x + 2} = 2x - 7 + \dfrac{19}{x + 2}$

$$\begin{array}{r|rrr} -2 & 2 & -3 & 5 \\ & \downarrow & -4 & 14 \\ \hline & 2 & -7 & 19 \\ & \uparrow & & \\ & x & & \end{array}$$

11. $\dfrac{x^3 - 1}{x - 1} = \dfrac{x^3 + 0x^2 + 0x - 1}{x - 1} = x^2 + x + 1$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & -1 \\ & \downarrow & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 0 \\ & \uparrow & \uparrow & & \\ & x^2 & x & & \end{array}$$

18. $\dfrac{2a^4 + 6a^3 + a^2 + 2a - 5}{a + 3} = 2a^3 + a - 1 - \dfrac{2}{a + 3}$

$$\begin{array}{r|rrrrr} -3 & 2 & 6 & 1 & 2 & -5 \\ & \downarrow & -6 & 0 & -3 & 3 \\ \hline & 2 & 0 & 1 & -1 & -2 \\ & \uparrow & \uparrow & \uparrow & & \\ & a^3 & a^2 & a & & \end{array}$$

23.
$$\begin{array}{r|rrrrr} -3 & 3 & -2 & 1 & -4 & 1 \\ & \downarrow & -9 & 33 & -102 & 318 \\ \hline & 3 & -11 & 34 & -106 & 319 \end{array}$$

$P(-3) = 319$

28.
$$\begin{array}{r|rrrrrrr} -2 & 8 & 0 & 0 & 0 & 3 & -2 & -5 \\ & \downarrow & -16 & 32 & -64 & 128 & -262 & 528 \\ \hline & 8 & -16 & 32 & -64 & 131 & -264 & 523 \end{array}$$

$P(-2) = 523$

35.
$$\begin{array}{r|rrrrr} -1 & 1 & -9 & 18 & 0 & -3 \\ & \downarrow & -1 & 10 & -28 & 28 \\ \hline & 1 & -10 & 28 & -28 & 25 \leftarrow x + 1 \text{ is } \textit{not} \text{ a factor} \end{array}$$

40.
$$\begin{array}{r|rrrr} -2 & 1 & 7 & 4 & -12 \\ & \downarrow & -2 & -10 & 12 \\ \hline & 1 & 5 & -6 & 0 \end{array}$$

set $x^2 + 5x - 6 = 0$

$(x + 6)(x - 1) = 0$

$x + 6 = 0$ or $x - 1 = 0$

$x = -6$ $\quad x = 1$

The solution set is $\{-6, -2, 1\}$.

45.
$$\begin{array}{r} x^2 + 2x + 1 \\ x^2 - 2x + 1 \overline{\smash{)}\, x^4 + 0x^3 - 2x^2 + 0x + 1} \\ \underline{x^4 - 2x^3 + x^2 \qquad\qquad} \\ 2x^3 - 3x^2 + 0x \quad \\ \underline{2x^3 - 4x^2 + 2x} \quad \\ x^2 - 2x + 1 \\ \underline{x^2 - 2x + 1} \\ 0 \end{array}$$

$x^2 + 2x + 1 = 0$

$(x + 1)^2 = 0$

$x + 1 = 0$

$x = -1$

The solution set is $\{-1, 1\}$.

47. Since $2, -1$ and 4 are zeros, then

$P(x) = (x - 2)(x + 1)(x - 4) = (x^2 - x - 2)(x - 4)$

$= x^3 - 5x^2 + 2x + 8$

50. Since $-2, 2, 3$ and -3 are zeros, then

$P(x) = (x + 2)(x - 2)(x - 3)(x + 3)$

$= (x^2 - 4)(x^2 - 9) = x^4 - 13x^2 + 36$

55. Since -1 of multiplicity 2 and 4 of multiplicity 2 are zeros, then

$P(x) = (x + 1)^2(x - 4)^2 = (x^2 + 2x + 1)(x^2 - 8x + 16)$

$= x^4 - 6x^3 + x^2 + 24x + 16$

Review exercises

1. $\emptyset$ **2.** $\{24\}$ **3.** $\{4\}$ **4.** 9 and 36 **5.** $R = \dfrac{3V + \pi h^3}{3\pi h^2}$

Exercise 4–7

Answers to odd-numbered problems

1. $\{-21\}$ **3.** $\left\{\frac{-15}{11}\right\}$ **5.** $\left\{\frac{10}{3}\right\}$ **7.** $\left\{\frac{67}{7}\right\}$ **9.** $\{12\}$

11. $\left\{\frac{-13}{6}\right\}$ **13.** $\emptyset$ **15.** $\left\{\frac{-3}{5}\right\}$ **17.** $\left\{\frac{-13}{5}\right\}$

19. $\emptyset$; 2 is extraneous **21.** $\left\{\frac{13}{3}\right\}$ **23.** $\left\{\frac{-16}{5}\right\}$ **25.** $\left\{\frac{13}{2}\right\}$

27. $\emptyset$; 3 is extraneous **29.** $\{-15\}$ **31.** $\left\{\frac{-17}{3}\right\}$ **33.** $\{26\}$

35. $\left\{\frac{-31}{5}\right\}$ **37.** $b = \dfrac{3a}{4 - 7a}$ **39.** $p = \dfrac{3q - 3}{3a + 4}$

41. $y = \dfrac{5x + 19}{3}$ **43.** $r = \dfrac{S - a}{S}$ **45.** $b_1 = \dfrac{2A - b_2 h}{h}$

47. $t = \dfrac{L_t - L_0}{kL_0}$; $L_0 = \dfrac{L_t}{kt + 1}$ **49.** $t = \dfrac{I}{Pr}$

51. $R_3 = \dfrac{RR_1R_2}{R_1R_2 - RR_2 - RR_1}$ **53.** $A = \dfrac{aF}{f}$

55. $T_2 = \dfrac{R_2M + R_2T_1 - R_1M}{R_1}$; $R_1 = \dfrac{R_2(M + T_1)}{M + T_2}$ 57. a. $\dfrac{1}{10}$ b. $\dfrac{1}{2}$ c. $\dfrac{t}{10}$ 59. a. $\dfrac{200}{5} = 40$ mph b. $\dfrac{200}{4} = 50$ mph c. $\dfrac{200}{t}$ mph 61. a. $\dfrac{1}{n}, \dfrac{3}{n}$ b. $\dfrac{15}{n}$ 63. $\dfrac{5 + n}{6 - n}$

Solutions to trial exercise problems

5. $\dfrac{4}{a} - \dfrac{6}{3a} = \dfrac{3}{5}$

Multiply by the LCM, $15a$.

$$\begin{aligned} 15a \cdot \frac{4}{a} - 15a \cdot \frac{6}{3a} &= 15a \cdot \frac{3}{5} \\ 15 \cdot 4 - 5 \cdot 6 &= 3a \cdot 3 \\ 60 - 30 &= 9a \\ 30 &= 9a \\ \frac{10}{3} &= a \end{aligned}$$

The solution set is $\left\{\dfrac{10}{3}\right\}$.

16. $1 + \dfrac{5}{3m - 9} = \dfrac{10}{m - 3}$

Multiply by the LCM, $3(m - 3)$.

$$\begin{aligned} 3(m-3) \cdot 1 + 3(m-3) \cdot \frac{5}{3(m-3)} &= 3(m-3) \cdot \frac{10}{m-3} \\ 3m - 9 + 5 &= 3 \cdot 10 \\ 3m - 4 &= 30 \\ 3m &= 34 \\ m &= \frac{34}{3} \end{aligned}$$

The solution set is $\left\{\dfrac{34}{3}\right\}$.

21. $4 - \dfrac{2x}{5 - x} = \dfrac{6}{x - 5}$

Since $5 - x = -(x - 5)$, then we have

$4 - \dfrac{2x}{-(x - 5)} = \dfrac{6}{x - 5}$

$4 + \dfrac{2x}{x - 5} = \dfrac{6}{x - 5}$. Multiply by $x - 5$.

$$\begin{aligned} 4(x-5) + (x-5) \cdot \frac{2x}{x-5} &= (x-5) \cdot \frac{6}{x-5} \\ 4x - 20 + 2x &= 6 \\ 6x - 20 &= 6 \\ 6x &= 26 \\ x &= \frac{26}{6} = \frac{13}{3} \end{aligned}$$

The solution set is $\left\{\dfrac{13}{3}\right\}$.

29. $\dfrac{6}{q^2 + q - 6} = \dfrac{5}{q^2 + 3q - 10}$

Since $q^2 + q - 6 = (q + 3)(q - 2)$ and $q^2 + 3q - 10 = (q + 5)(q - 2)$, the LCM is $(q + 3)(q - 2)(q + 5)$.

Multiply each member by the LCM.

$$\begin{aligned} (q+3)(q-2)(q+5) \cdot \frac{6}{(q+3)(q-2)} &= (q+3)(q-2)(q+5) \cdot \frac{5}{(q+5)(q-2)} \\ (q+5) \cdot 6 &= (q+3) \cdot 5 \\ 6q + 30 &= 5q + 15 \\ q &= -15 \end{aligned}$$

The solution set is $\{-15\}$.

41. $\dfrac{y - 3}{x + 2} = \dfrac{5}{3}$

Multiply by the LCM, $3(x + 2)$.

$$\begin{aligned} 3(x+2) \cdot \frac{y-3}{x+2} &= 3(x+2) \cdot \frac{5}{3} \\ 3(y-3) &= (x+2) \cdot 5 \\ 3y - 9 &= 5x + 10 \\ 3y &= 5x + 19 \\ y &= \frac{5x + 19}{3} \end{aligned}$$

48. $R_x = R_m\left(\dfrac{E_1}{E_2} - 1\right)$

Now $R_x = R_m \cdot \dfrac{E_1}{E_2} - R_m$

$R_x + R_m = \dfrac{R_mE_1}{E_2}$

Multiply each member by E_2.

$E_2(R_x + R_m) = R_mE_1$

Divide each member by R_m.

$\dfrac{E_2(R_x + R_m)}{R_m} = E_1$

57. If Ann can paint the house in 10 hours, she can paint a. $1 \div 10 = \dfrac{1}{10}$, b. $5 \div 10 = \dfrac{5}{10} = \dfrac{1}{2}$, c. $t \div 10 = \dfrac{t}{10}$ of the house in 1 hour, 5 hours, and t hours. 61. Let n = 1st number, then $\dfrac{1}{3}n = \dfrac{n}{3}$ = 2nd number. a. The reciprocals are $\dfrac{1}{n}$ and $\dfrac{3}{n}$. b. Five times the 2nd reciprocal $5 \cdot \dfrac{3}{n} = \dfrac{15}{n}$.

Review exercises

1. $(2p + 5q)(2p - 5q)$ 2. $(x - 12)^2$ 3. $(2y + 1)(y - 8)$
4. \$12,000 at 6%; \$8,000 at 8% 5. 8 6. $P(-3) = 12$

Exercise 4–8

Answers to odd-numbered problems

1. 1 hour 12 minutes 3. 40 minutes 5. 45 hours
7. $13\frac{11}{13}$ hours $\approx$ 13.8 hours 9. 18 hours for pump B; 36 hours for pump A 11. 7 hours 13. $\dfrac{48}{17}$ mph 15. 30 km/hr
17. 1,600 miles 19. $1\frac{2}{3}$ miles 21. $\dfrac{2}{3}$ and 2 23. 13
25. 3 27. 3 mph 29. 6 hours 31. 30 mph
33. Sarah, 4 mph; Erin, 6 mph

Solutions to trial exercise problems

4. Let x = minutes taken to mix the 12 loaves working together.

$\frac{1}{36} + \frac{1}{40} + \frac{1}{30} = \frac{1}{x}$ Multiply by the LCM, $360x$.

$$360x \cdot \frac{1}{36} + 360x \cdot \frac{1}{40} + 360x \cdot \frac{1}{30} = 360x \cdot \frac{1}{x}$$
$$10x + 9x + 12x = 360$$
$$31x = 360$$
$$x = \frac{360}{31} = 11\frac{19}{31}$$

Working together, they could mix the 12 loaves in $11\frac{19}{31}$ minutes ≈ 11.6 minutes.

7. Let x = number of hours necessary to fill the basin with all pipes open.

$\frac{1}{10} + \frac{1}{12} - \frac{1}{9} = \frac{1}{x}$ Multiply by the LCM, $180x$.

$$180x \cdot \frac{1}{10} + 180x \cdot \frac{1}{12} - 180x \cdot \frac{1}{9} = 180x \cdot \frac{1}{x}$$
$$18x + 15x - 20x = 180$$
$$13x = 180$$
$$x = \frac{180}{13} = 13\frac{11}{13}$$

It would take $13\frac{11}{13}$ hours ≈ 13.8 hours to fill the basin with all three pipes open.

10. Let x = time for slower microprocessor to do the job. Then $\frac{3}{5}x$ = time for faster microprocessor to do the job.

$$\frac{1}{x} + \frac{1}{\frac{3}{5}x} = \frac{1}{2}$$
$$\frac{1}{x} + \frac{5}{3x} = \frac{1}{2}$$
$$6x \cdot \frac{1}{x} + 6x \cdot \frac{5}{3x} = 6x \cdot \frac{1}{2}$$
$$6 + 2 \cdot 5 = 3x$$
$$16 = 3x$$
$$x = \frac{16}{3}$$
$$\frac{3}{5}x = \frac{16}{5}$$

It would take the microprocessors $\frac{16}{3}$ or $5\frac{1}{3}$ milliseconds and $\frac{16}{5}$ or $3\frac{1}{5}$ milliseconds, respectively, to process the set of inputs individually.

14. Let r = speed of the wind. Then $300 + r$ = speed of the plane with the wind and $300 - r$ = speed of the plane against the wind.

Now t_w (time with the wind) $= \frac{950}{300 + r}$

t_a (time against the wind) $= \frac{650}{300 - r}$

Then $\frac{950}{300 + r} = \frac{650}{300 - r}$.

Multiply each member by $(300 + r)(300 - r)$.

$$(300 + r)(300 - r) \cdot \frac{950}{300 + r} = (300 + r)(300 - r) \cdot \frac{650}{300 - r}$$
$$(300 - r) \cdot 950 = (300 + r) \cdot 650$$
$$285{,}000 - 950r = 195{,}000 + 650r$$
$$90{,}000 = 1{,}600r$$
$$r = \frac{90{,}000}{1{,}600}$$
$$r = \frac{900}{16} = 56\frac{1}{4}$$

The speed of the wind is $56\frac{1}{4}$ mph.

21. Let n = one number, then $3n$ = the other number. The reciprocals are $\frac{1}{n}$ and $\frac{1}{3n}$.

sum of the reciprocals is 2

$$\frac{1}{n} + \frac{1}{3n} = 2$$
$$3n \cdot \frac{1}{n} + 3n \cdot \frac{1}{3n} = 3n \cdot 2$$
$$3 + 1 = 6n$$
$$4 = 6n$$
$$n = \frac{4}{6} = \frac{2}{3}$$
$$3n = 3 \cdot \frac{2}{3} = 2$$

The numbers are $\frac{2}{3}$ and 2.

27. Let x = speed of current.

	d	r	t
downstream	10	$12 + x$	$\frac{10}{12 + x}$
upstream	6	$12 - x$	$\frac{6}{12 - x}$

Since times are the same,

$$\frac{10}{12 + x} = \frac{6}{12 - x}$$
$$10(12 - x) = 6(12 + x)$$
$$120 - 10x = 72 + 6x$$
$$16x = 48$$
$$x = 3$$

The current has a speed of 3 mph.

30. Inlet pipe fills $\frac{1}{45}$ of the tank in 1 minute

Outlet pipe empties $\frac{1}{30}$ of the tank in 1 minute.

Let x = number of minutes to empty the tank

$$\frac{1}{30} - \frac{1}{45} = \frac{1}{x}$$

$$90x \cdot \frac{1}{30} - 90x \cdot \frac{1}{45} = 90x \cdot \frac{1}{x}$$

$$3x - 2x = 90$$

$$x = 90$$

It would take 90 minutes to empty the tank.

Review exercises

1. $8x^3 - 12x^2 + 4x$ **2.** $3x^2 - 16x + 5$ **3.** $25z^2 - 40z + 16$ **4.** $4y^2 - 9$ **5.** $\frac{1}{y}$ **6.** x^4y^3 **7.** -7 or 7 **8.** -5

Chapter 4 review

1. $\{x|x \in R, x \neq -7\}$ **2.** $\left\{x|x \in R, x \neq \frac{4}{3}\right\}$ **3.** $\{x|x \in R, x \neq 5\}$ **4.** $\left\{a|a \in R, a \neq -\frac{4}{3},\frac{4}{3}\right\}$ **5.** $\left\{z|z \in R, z \neq -\frac{5}{2},\frac{1}{3}\right\}$ **6.** $\left\{y|y \in R, y \neq \frac{2}{3}\right\}$ **7.** $\frac{ab^3}{c^2}$ **8.** $\frac{-2n^2}{7mp^4}$ **9.** $\frac{5}{6}$ **10.** $\frac{3}{a-2}$ **11.** $\frac{-5}{2y+x}$ **12.** $\frac{y^2+4y+16}{y+4}$ **13.** $\frac{a-12}{a+1}$ **14.** $\frac{4x+3}{5x-1}$ **15.** $\frac{-(2y+3)}{2(3y+2)}$ **16.** $\frac{6y}{x}(x \neq 0, y \neq 0)$ **17.** $12ay(a \neq 0, y \neq 0)$ **18.** $\frac{(4p+3)(p-4)}{3}\left(p \neq -4,\frac{3}{4}\right)$ **19.** $\frac{z-3}{2(z+1)(z-1)}(z \neq -1,1,3)$ **20.** $\frac{(m^2+2m+4)(m+6)}{m(m+5)}(m \neq -5,0,2,3)$ **21.** $1\left(a \neq -7,-3,-\frac{2}{5},\frac{1}{2}\right)$ **22.** $\frac{(x+7)(2x-1)}{(4x^2-2x+1)(x+2)}\left(x \neq -2,-\frac{1}{2},\frac{1}{2},7\right)$ **23.** $\frac{x^2}{(4x+5)^2}\left(x \neq -\frac{5}{4},\frac{5}{4}\right)$ **24.** $\frac{y+3}{y+2}\left(y \neq -3,-2,\frac{4}{7}\right)$ **25.** $\frac{m-n}{m+n}\left(m \neq -n,\frac{n}{2}; q \neq -p,p\right)$ **26.** $180x^3y^3$ **27.** $6x^2(x+2)(x+4)(x-4)$ **28.** $3a(a+5)(a-2)$ **29.** $p(p-5)(p+5)^2$ **30.** $\frac{41x}{12y}$ **31.** $\frac{-2n^2-28n-25}{(n+4)(n-1)}$ **32.** $\frac{10p^2+29p+81}{p(p+9)(p+2)(p-2)}$ **33.** $\frac{6b^2+16b-23}{3b-2}$ **34.** $\frac{2y^2+18y+3}{(y+7)(y-7)}$ **35.** $\frac{-(4x^2+15x+4)}{(x-6)(x+6)(x+4)}$ **36.** $\frac{37}{2(a-2)}$ **37.** $\frac{-5x^2+63x-102}{8(x-7)(x+4)}$ **38.** $\frac{4a+b}{(a-2b)(a+2b)}$ **39.** $\frac{1}{R_t} = \frac{I_1E_2E_3 + I_2E_1E_3 + I_3E_1E_2}{E_1E_2E_3}$ **40.** $\frac{2}{a}$ **41.** $\frac{5x}{4x-12}$ **42.** $\frac{3x+6}{x-5}$ **43.** $\frac{7b-38}{8b-34}$ **44.** $\frac{x^2y-xy^2}{2y+3x}$ **45.** $\frac{p^2-4p+5}{p^2-4}$ **46.** $5a^6+3a^2+2$ **47.** $6a^3b^2c^2-3c^3+1$ **48.** $3x^2-3x+4$ **49.** x^3+x^2-x+1 **50.** $P(-2) = -35$ **51.** $P(1) = 11$ **52.** $P(-1) = -12$ **53.** -3 is a solution **54.** -1 is not a solution **55.** 2 is not a solution **56.** $\left\{\frac{55}{216}\right\}$ **57.** $\{66\}$ **58.** $\left\{-\frac{37}{6}\right\}$ **59.** $\left\{\frac{7}{20}\right\}$ **60.** $\left\{-\frac{3}{29}\right\}$ **61.** $p = \frac{4m-6n+26}{3}$ **62.** $C = \frac{5}{9}(F-32)$ **63.** $V_1 = \frac{P_2V_2T_1}{P_1T_2}$ **64.** $R_2 = \frac{R_tR_1}{R_1-R_t}$ **65.** 2 days **66.** 60 mph, automobile; 90 mph, train **67.** 3 mph **68.** $-\frac{1}{28}$

Chapter 4 cumulative test

1. 25 **2.** -7 **3.** $\frac{4}{15}$ **4.** 5 **5.** $\frac{13}{30}$ **6.** $7x+11$ **7.** $-24a^6b^5$ **8.** $x^3+2x^2-3x+20$ **9.** 2 **10.** $\frac{x^2+6x-16}{x^2+3x}$ **11.** $\frac{-10y-13}{24}$ **12.** $10x^2+39x-27$ **13.** $16x^2-40x+25$ **14.** $9y^2-25$ **15.** $2x^2+9x+27+\frac{80}{x-3}$ **16.** $\frac{6a^2+11a-10}{4a^2+23a-35}$ **17.** $\frac{-2x+19}{(2x-3)(x+1)(2x+1)}$ **18.** $\left\{y|y \in R, y \neq \frac{3}{2}\right\}$ **19.** $\left\{x|x \in R, x \neq \frac{1}{2},-\frac{1}{2}\right\}$ **20.** $\{x|x \in R, x \neq -5,5\}$ **21.** $\{14\}$ **22.** $\left\{-\frac{11}{24}\right\}$ **23.** $\left\{-\frac{1}{8}\right\}$ **24.** $-\frac{13}{14}$ **25.** $\frac{4x^2-3x-10}{5x^2+2x-16}$ **26.** $\{x|x \leq -1\}$ **27.** $\left\{y|y < \frac{17}{2}\right\}$ **28.** $\{z|z > -5\}$ **29.** $\left\{x|x \geq \frac{97}{23}\right\}$ **30.** $-\frac{3a}{2b^2}$ **31.** $\frac{p+4}{p+3}$ **32.** $-\frac{3}{2}$ **33.** $\left\{-\frac{1}{9}\right\}$ **34.** $\frac{1}{2}$ or 6 **35.** $P(-2) = 27$

Chapter 5

Exercise 5-1

Answers to odd-numbered problems

1. 4.243 **3.** -5.745 **5.** $\sqrt[3]{9}$ **7.** $\sqrt{x}$ **9.** $\sqrt[5]{b^4}$ **11.** 4 **13.** 16 **15.** 27 **17.** 64 **19.** $\frac{1}{2}$ **21.** $\frac{1}{2}$ **23.** $\frac{1}{9}$ **25.** $\frac{-1}{8}$ **27.** $\frac{1}{\sqrt[4]{x^3}}$ **29.** $a^{4/7}$ **31.** $x^{1/5}$ **33.** -8 **35.** $|-4| = 4$ **37.** $|2x-y|$ **39.** 256 **41.** $E = \frac{T}{\sqrt{(x^2+r^2)^3}}$ **43.** 24 miles per hour

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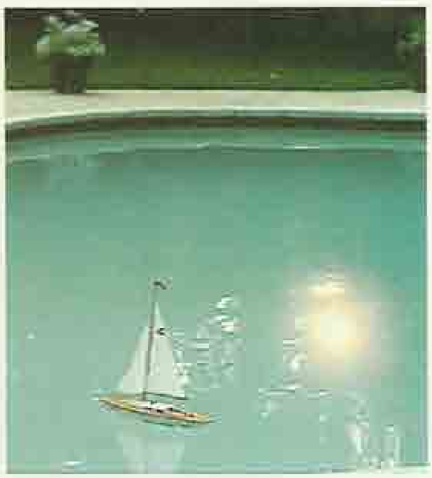
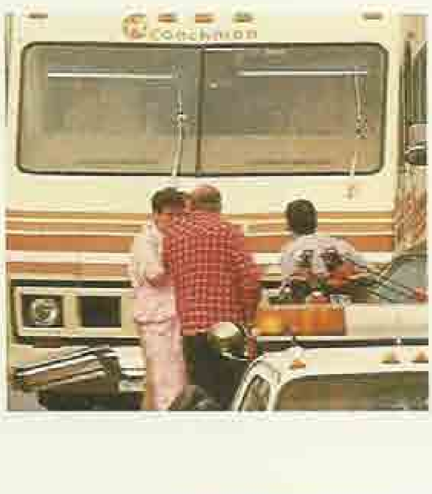


Chapter 5 ■ Exponents, Roots, and Radicals



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